Block 4/5

Quad Two Column Proofs

Let's recap what we've learned so far this year that pertains to proofs:

Vocabulary:

- midpoint
- median (2-step)
- segment bisector
- angle bisector
- ∥ lines
- ⊥ lines
- altitude (2-step)
- ⊥ bisector

Common Cycles:

- Segment Subtraction/Addition
- Angle Subtraction/Addition
- Congruent Supplements
- Congruent Segments/Angles being bisected.

If two parallel lines are crossed by a transversal, then…

- alt int. ∠'s are ≅
- alt ext ∠'s are ≅
- corresponding ∠'s are ≅
- same-side interior are supp
- same side exterior are supp.

Freebies:

- Vertical ∠'s are ≅
- Linear pairs of ∠'s are supp.
- Reflexive Property

5 Ways to Prove △'s ≅ :

SSS
SAS
AAS
ASA
HL

C
P
C
T
C

Any of the properties of quadrilaterals that we have learned can be used in a two column proof. For example:

"If quadrilateral ABCD is a parallelogram, then 1 pair opp sides both ∥ " and

"If quadrilateral BEST is a square, then diagonals are ⊥ "

"If quadrilateral SOME has two sets of opposite sides parallel, then its a parallelogram "

"If parallelogram GIRL has two consecutive sides congruent, then its a rhombus "

There are three different types of proof problems you could face:

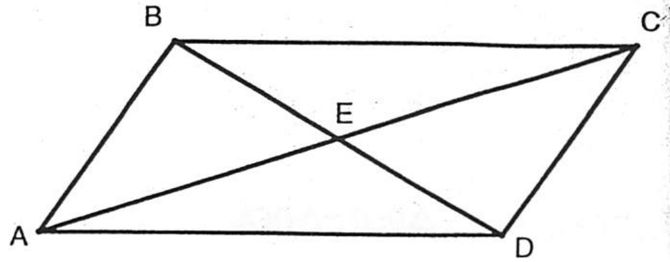
1) Given: parts
   Prove: figure is a certain quadrilateral

2) Given: a quadrilateral (use its properties)
   Prove: triangles ≅

3) Given: a quadrilateral (use its properties)
   Prove: parts

1) **Given:** $\overline{AB} \cong \overline{CD}$
$\angle CAB \cong \angle ACD$

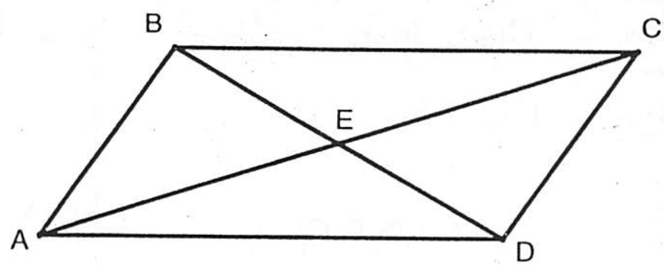
Prove: *ABCD* is a parallelogram



Statements	Reasons
(1) $\overline{AB} \cong \overline{CD}$ $\angle CAB \cong \angle ACD$	(1) Given
(2) $\overline{AB} \parallel \overline{CD}$	(2) If 2 lines are cut by a transversal and altr int ∠'s are ≅ then the lines are ∥.
(3) ABCD is a ∥ogram	(3) If quad ABCD has 1 pair opp sides ≅ and ∥ then its a ∥ogram.

2 **Given:** $\overline{AE}$ is a median to $\triangle ABD$
E is the midpoint of $\overline{AC}$

Prove: *ABCD* is a parallelogram



Statements	Reasons
(1) $\overline{AE}$ is a median to $\triangle ABD$ E is midpt of $\overline{AC}$	(1) Given
(2) E is midpt of $\overline{BD}$	(2) A median is drawn from the vertex to the midpt of opp side.
(3) $\overline{AC}$ and $\overline{BD}$ bisect each other	(3) If 2 seg have the same midpt. then the 2 segments bisect each other
(4) ABCD is a ∥ogram	(4) If quad ABCD has diags that bisect each other then its a ∥ ogram.

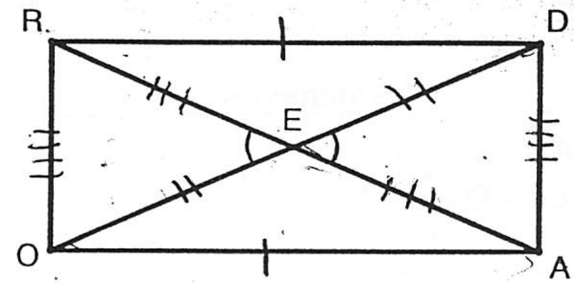
3)

Given: $\triangle DER \cong \triangle OEA$

$\triangle DER$ is isosceles with $\overline{DE} \cong \overline{RE}$

$\triangle OEA$ is isosceles with $\overline{OE} \cong \overline{AE}$

Prove: *ROAD* is a rectangle

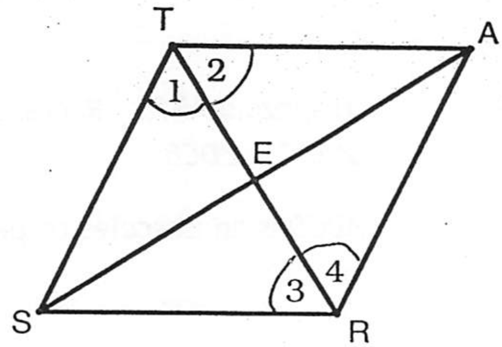


Statements	Reasons
1) $\Delta DER \cong \Delta OEA$ ΔDER is isos w/ $\overline{DE} \cong RE$ ΔOEA is isos w/ $\overline{OE} \cong \overline{AE}$	1) Given
2) $\overline{OA} \cong \overline{DR}$, $\overline{OE} \cong \overline{DE}$ $\overline{AE} \cong \overline{RE}$	2) CPCTC
3) $\angle REO \cong \angle AED$	3) Vertical $\angle$'s are $\cong$
4) $\Delta DEA \cong \Delta OER$	4) SAS
5) $\overline{OR} \cong \overline{DA}$	5) CPCTC
6) ROAD is a parallelogram	6) If quad ROAD ~~has~~ has 2 pair opp sides $\cong$ then its a llogram
7) $\underbrace{\overline{RE} + \overline{AE}} \cong \overline{DE} + \overline{OE}$	7) ~~[illegible]~~ Addition Prop of Equality
8) $\overline{RA} \cong \overline{OD}$	8) Segment Addition Postulate
9) ROAD is a rectangle	9) If ~~a~~ llogram ROAD had $\cong$ diags. then its a rectangle.

9

Given: $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$

Prove: *STAR* is a rhombus

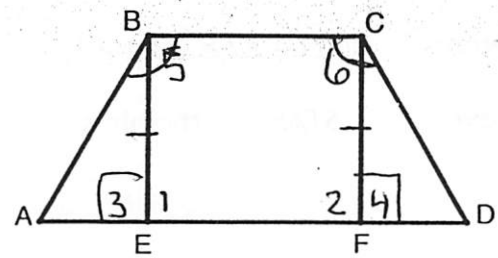


Statements	Reasons
(1) $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$	(1) Given
(2) $\overline{ST} \parallel \overline{RA}$, $\overline{TA} \parallel \overline{SR}$	(2) If ~~[illegible]~~ 2 lines are cut by a transversal and alt. int ∠'s are ≅ then the lines are ∥
(3) STAR is a ∥ogram	(3) If opp sides are ∥ then STAR is a parallelogram
(4) ~~[illegible]~~ $\overline{TS} \cong \overline{RS}$	(4) ~~[illegible]~~ If 2 ∠'s of a Δ are congruent, sides opp those ∠'s are ≅
(5) STAR is a rhombus	(5) If STAR ∥ogram has consecutive sides ≅ then its a rhombus.

5)

Given: Trapezoid *ABCD*, Rectangle EBCF
$\angle ABC \cong \angle DCB$

Prove: *ABCD* is an isosceles trapezoid

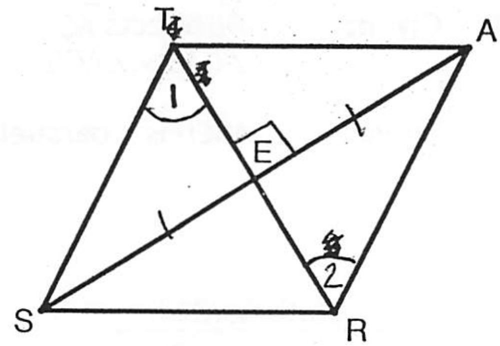


Statements	Reasons
① Trap ABCD, Rect EBCF $\angle ABC \cong \angle DCB$	① Given
② $\overline{BE} \cong \overline{CF}$	② If EBCF is a rectangle then opp sides are $\cong$
③ $\angle 1 \cong \angle 2$, $\angle 5 \cong \angle 6$	③ If EBCF is a rectangle then all $\angle$'s are $\cong$ 90° $\angle$'s
④ $\angle 1$ and $\angle 3$ are supp $\angle 2$ and $\angle 4$ are supp	④ Linear Pairs are Supp.
⑤ $\angle 3 \cong \angle 4$	⑤ If 2 $\angle$'s are supplements to 2 $\cong$ $\angle$'s then those $\angle$'s are $\cong$
⑥ $\angle ABC - \angle 5 \cong \angle DCB - \angle 6$	⑥ Subtraction prop of equality
⑦ $\angle ABE \cong \angle DCF$	⑦ Angle subtraction Postulate
⑧ $\triangle ABE \cong \triangle DCF$	⑧ ASA
⑨ $\overline{AB} \cong \overline{CD}$	⑨ CPCTC
⑩ ABCD is an isos trap.	⑩ If a trap has non-∥ sides $\cong$ then its isos.

6

Given: $\angle TEA$ is a right angle, $\angle 1 \cong \angle 2$
E is the midpt of $\overline{SA}$

Prove: $STAR$ is a rhombus

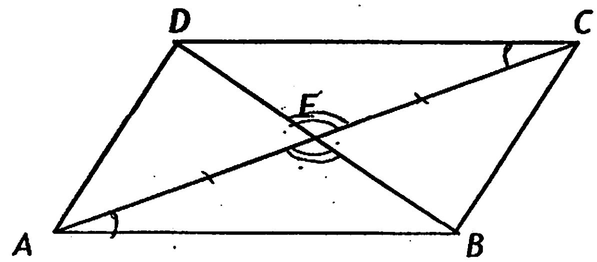


Statements	Reasons
(1) $\angle TEA$ is right $\angle 1 \cong \angle 2$ E is midpt of $\overline{SA}$	(1) Given
(2) $\overline{ST} \parallel \overline{AR}$	(2) If 2 lines cut by a transversal and alt int $\angle$'s are $\cong$ then lines are $\parallel$
(3) $\overline{SE} \cong \overline{AE}$	(3) If midpt given 2 $\cong$ seg formed.
(4) $\angle SET \cong \angle AER$	(4) Vertical $\angle$'s $\cong$
(5) $\triangle SET \cong \triangle AER$	(5) AAS
(6) $\overline{ST} \cong \overline{AR}$	(6) CPCTC
(7) STAR is $\parallel$ogram	(7) If quad STAR has 1 pair opp sides $\cong$ and $\parallel$ then its a $\parallel$ogram
(8) $\overline{TR} \perp \overline{SA}$	(8) If segments intersect to form a right $\angle$ then the segments are $\perp$
(9) STAR is a rhombus	(9) If $\parallel$gram STAR has $\perp$ diags then its a rhombus.

7 **Given:** $\overline{DB}$ bisects $\overline{AC}$

$\angle CAB \cong \angle ACD$

Prove: *ABCD* is a parallelogram



Statements	Reasons
(1) $\overline{DB}$ bisects $\overline{AC}$ $\angle CAB \cong \angle ACD$	(1) Given
(2) $\overline{AE} \cong \overline{CE}$	(2) If seg is bisected then 2 $\cong$ seg. formed
(3) $\angle DEC \cong \angle BEA$	(3) Vertical $\angle$'s are $\cong$
(4) $\triangle AEB \cong \triangle CED$	(4) ASA
(5) $\overline{BA} \cong \overline{DC}$	(5) CPCTC
(6) $\overline{BA} \parallel \overline{DC}$	(6) If 2 lines are cut by a transversal and alt int $\angle$'s are $\cong$ then the 2 lines are $\parallel$
(7) ABCD is a parallelogram	(7) If a quad has 1 pair opp sides $\cong$ and $\parallel$ then the quad is a parallelogram

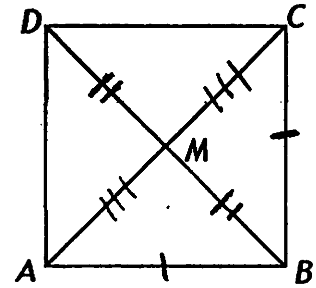
8)

Given: $\triangle ABC$ is an isosceles right triangle with $\overline{BA} \cong \overline{BC}$

$\overline{BM}$ is a median of $\triangle ABC$

$\overline{BM} \cong \overline{DM}$

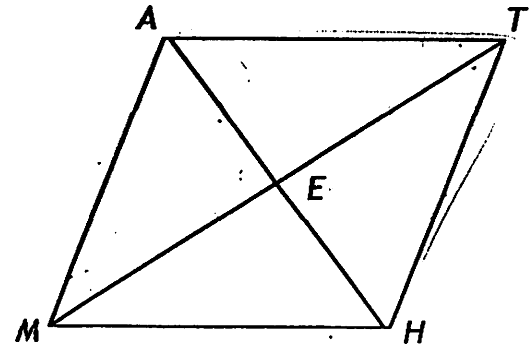
Prove: *ABCD* is a square



Statements	Reasons
① $\triangle ABC$ is isos rt $\triangle$ w/ $\overline{BA} \cong \overline{BC}$; $\overline{BM}$ is a median of $\triangle ABC$; $\overline{BM} = \overline{DM}$	① Given
② M is midpt of AC	② A median is drawn to the midpt of the opp side.
③ $\overline{CM} \cong \overline{AM}$	③ If a midpt is drawn then 2 $\cong$ seg are formed.
④ ABCD is a parallelogram	④ If a quad has diagonals that bisect each other then it is a parallelogram
⑤ ABCD is a rectangle	⑤ If a parallelogram has a rt $\angle$ then it is a rectangle.
⑥ ABCD is a rhombus	⑥ If a parallelogram has 2 consecutive sides $\cong$ then it is a rhombus
⑦ ABCD is a square	⑦ If a quad is a parallelogram, a rectangle and a rhombus then it must be a square.

9 [redacted]) **Given:** Quadrilateral *MATH* is a rhombus

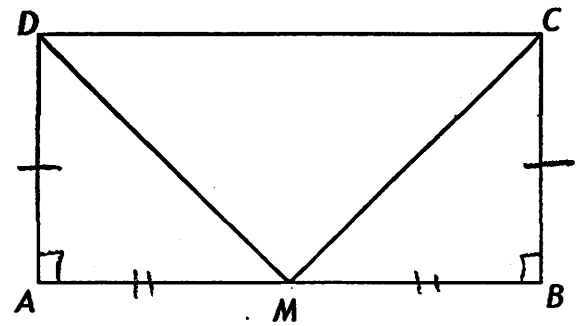
Prove: $\Delta MAE \cong \Delta THE$



Statements	Reasons
① Quad MATH is rhombus	① Given
② $\overline{MA} \cong \overline{TH}$	② If quad is a rhombus then opposite sides are ≅
③ $\overline{AE} \cong \overline{HE}$, $\overline{ME} \cong \overline{TE}$	③ If quad is a rhombus then diagonals bisect each other
④ $\Delta MAE \cong \Delta THE$	④ SSS

10 9) **Given:** Rectangle *ABCD*
M is the midpoint of $\overline{AB}$

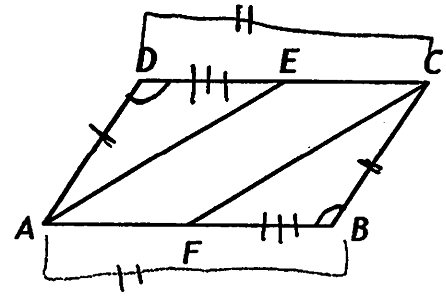
Prove: $\overline{CM} \cong \overline{DM}$



Statements	Reasons
① Rectangle ABCD M is midpt of $\overline{AB}$	① Given
② $\overline{AD} \cong \overline{CB}$	② If a quad is a rectangle then opp sides are $\cong$
③ $\angle A \cong \angle B$	③ If a quad is a rectangle then all $\angle$'s are $\cong$ 90° $\angle$'s
④ $\overline{AM} \cong \overline{BM}$	④ If a midpt is drawn then 2 $\cong$ seg formed
⑤ $\triangle ADM \cong \triangle BCM$	⑤ SAS
⑥ $\overline{CM} \cong \overline{DM}$	⑥ CPCTC

11 [redacted]) **Given:** *ABCD* is a parallelogram.
E is the midpoint of $\overline{DC}$.
F is the midpoint of $\overline{AB}$.

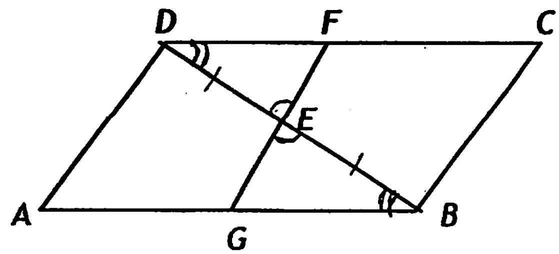
Prove: $\overline{AE} \cong \overline{CF}$



Statements	Reasons
① ABCD is a parallelogram E is midpt of $\overline{DC}$ F is midpt of $\overline{AB}$	① Given
② $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	② If a quad is a parallelogram then the quad has opp sides ≅
③ $\overline{DE} \cong \overline{BF}$	③ If a midpt is drawn in 2 ≅ seg then all seg formed are ≅.
④ ∠D ≅ ∠B	④ If a quad is a parallelogram then opp ∠'s are ≅
⑤ △ADE ≅ △CBF	⑤ SAS
⑥ $\overline{AE} \cong \overline{CF}$	⑥ CPCTC

12) . **Given:** *ABCD* is a parallelogram
$\overline{FG}$ bisects $\overline{DB}$.

Prove: $\overline{DB}$ bisects $\overline{FG}$

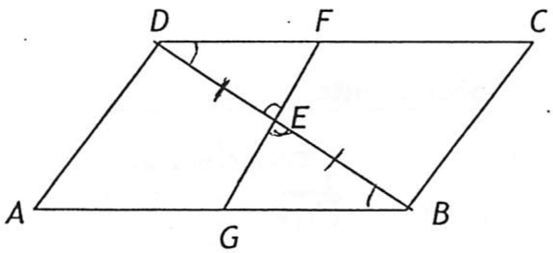


Statements	Reasons
(1) ABCD is a parallelogram $\overline{FG}$ bisects $\overline{DB}$	(1) Given
(2) $\angle DEF \cong \angle BEG$	(2) Vertical $\angle$'s are $\cong$
(3) $\overline{DC} \parallel \overline{BA}$	(3) If a quad is a parallelogram then opp sides are $\parallel$.
(4) $\angle FDE \cong \angle GBE$	(4) If $\parallel$ lines are cut by a transversal then alt int $\angle$'s are $\cong$
(5) $\triangle FDE \cong \triangle GBE$	(5) ASA
(6) $\overline{FE} \cong \overline{GE}$	(6) CPCTC
(7) E is midpt of $\overline{FG}$	(7) If 2 $\cong$ seg are formed out of a single seg then the midpt is the shared endpoint
(8) $\overline{DB}$ bisects $\overline{FG}$	(8) If a seg is drawn through the midpt of another seg then it is a seg bisector.

13)

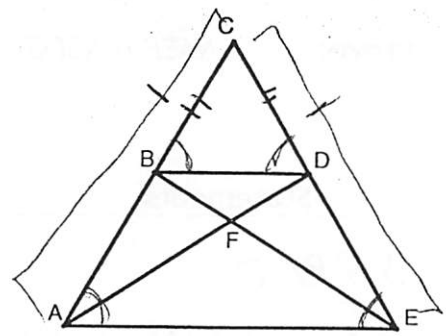
Given: *ABCD* is a parallelogram
$\overline{FG}$ bisects $\overline{DB}$.

Prove: $\overline{FE} \cong \overline{GE}$



Statements	Reasons
1 ABCD is a llogram $\overline{FG}$ bisects $\overline{DB}$	1 Given
2 $\overline{DE} \cong \overline{BE}$	2 If seg is bisected 2 ≅ segs are formed.
3 ∠DEF ≅ ∠BEG	3 Vertical ∠'s are ≅
4 $\overline{CD} \parallel \overline{AB}$	4 If ABCD is a llogram then opp sides are ∥
5 ∠FDE ≅ ∠GBE	5 If 2 ∥ lines are cut by a transversal then alt int ∠'s are ≅
6 △FDE ≅ △GBE	6 ASA
7 $\overline{FE} \cong \overline{GE}$	7 CPCTC

14)



Given: Isosceles $\triangle ACE$ with $\overline{AC} \cong \overline{EC}$

Trapezoid ABDE with $\overline{BD} \parallel \overline{AE}$

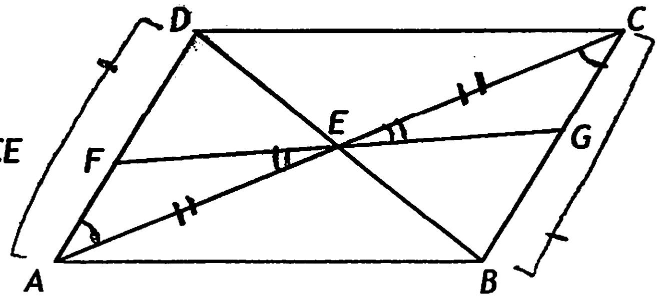
Prove: $\overline{AD} \cong \overline{EB}$

Statements	Reasons
1. Isos $\triangle ACE$ w/ $\overline{AC} \cong \overline{EC}$ Trap ABDE w/ $\overline{BD} \parallel \overline{AE}$	1. Given
2. $\angle CAE \cong \angle CEA$	2. Base $\angle$'s of an isos $\triangle$ are $\cong$
3. $\angle CBD \cong \angle CAE$ $\angle CDB \cong \angle CEA$	3. If $\parallel$ lines are cut by a transversal then corresponding $\angle$'s are $\cong$
4. $\angle CBD \cong \angle CDB$	4. Transitive Property
5. $\overline{CB} \cong \overline{CD}$	5. Sides opp $\cong$ $\angle$'s in a $\triangle$ are $\cong$
6. $\overline{AC} - \overline{BC} \cong \overline{EC} - \overline{DC}$	6. Subtraction Prop of Equality
7. $\overline{AB} \cong \overline{DE}$	7. Segment subtraction Postulate
8. ABDE is an isos trap.	8. If trap ABDE has non-$\parallel$ sides $\cong$ then its isos.
9. $\overline{AD} \cong \overline{EB}$	9. If ABDE is an isos trap then the diagonals are $\cong$

15 [redacted]) **Given:** Quadrilateral ABCD

$\overline{AD} \cong \overline{BC}$, $\angle DAE \cong \angle BCE$

Prove: $\Delta AEF \cong \Delta CEG$



Statements	Reasons
① Quad ABCD $\overline{AD} \cong \overline{BC}$ $\angle DAE \cong \angle BCE$	① Given
② $\overline{AD} \parallel \overline{BC}$	② If 2 lines are cut by a transversal and alt int ∠'s are ≅ then the 2 lines are ∥
③ ABCD is a parallelogram	③ If opp sides are ≅ and ∥ of a quad then its a parallelogram
④ $\overline{AE} \cong \overline{CE}$	④ If a parallelogram then diagonals bisect each other
⑤ $\angle AEF \cong \angle CEG$	⑤ Vertical ∠'s are ≅
⑥ $\Delta AEF \cong \Delta CEG$	⑥ ASA