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| **Perpendicular Bisectors**Produce**Circumcenter** | **Angle Bisectors**Produce**Incenter** |
| **Medians**Produce**Centroid** | **Altitudes**Produce**Orthocenter** |

**I. Midsegments of Triangles**

Using Geometer’s Sketchpad, we need to:

1. Construct and label ∆ABC.

2. Right-Click on each segment and select Thick.

3. Construct the midpoint D of $\overbar{AB}$, and midpoint E of $\overbar{AC}$.

4. Connect the midpoints, creating $\overbar{DE}$.

5. Measure the lengths of $\overbar{DE}$ and $\overbar{BC}$.

6. Click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice when you compare the lengths of $\overbar{DE}$ and $\overbar{BC}$?

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7. Click and Highlight $\overbar{BC}.$ From the **Measure** menu select Slope. Repeat with $\overbar{DE}.$

8. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice when you compare the slopes of $\overbar{DE}$ and $\overbar{BC}$?

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9. Measure  and . Then measure and 

10. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice when you compare these angle measures?

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11. Construct midpoint F on $\overbar{BC}$ and connect the midpoints to create ∆DEF.

12. Construct the Area of each of the four small triangles.

 - Click and Highlight each vertex of ∆DEF.
 - From the **Construct** menu, select Triangle Interior
 - From the **Measure** menu, select Area.
 - Repeat with ∆DEA, ∆EFC, and ∆DFB

13. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice about the areas of the four small triangles?

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14. Construct the Area of ∆ABC.

15. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice about the area of ∆ABC in comparison to each small triangle?

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16. Construct the Perimeter of each of the four small triangles.

 - Click and Highlight each vertex of ∆DEF.
 - From the **Construct** menu, select Triangle Interior
 - From the **Measure** menu, select Perimeter.
 - Repeat with ∆DEA, ∆EFC, and ∆DFB

17. Construct the Perimeter of ∆ABC.

What do you notice about the perimeter of ∆ABC in comparison to each small triangle?

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**II. Circumcenter of a Triangle**

Open a new Geometer’s Sketchpad Document

1. Construct and label ∆ABC. Right-click to make the segments thick.

2. Measure each angle of ∆ABC.

- Click and Highlight Point B, then Point A, then Point C. (The vertex for the angle you want to measure must be clicked 2nd.)
- From the **Measure** menu, select Angle.
- Repeat for  and 

3. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC to an acute scalene triangle.

4. We want to create the perpendicular bisector of each side of the triangle.

- Construct the midpoint D of $\overbar{AB}$.
- Click and Highlight $\overbar{AB}$ and point D.
- From the **Construct** menu, select Perpendicular Line.
- Repeat for midpoint E of $\overbar{AC}$ and midpoint E of $\overbar{BC}$.

What do you notice about the intersection of the three perpendicular bisectors?

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5. Place Point G at the intersection of the perpendicular bisectors. The point at which 3 or more lines intersect is called the **point of concurrency**.

6. Click on the **Circle Tool**. Click on Point G and extend the edge of the circle to one of the vertices of ∆ABC.

7. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

How would you describe the circle that you constructed?

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Where is the center of the circle located? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The point of concurrency of the Perpendicular Bisectors of a triangle is called the **Circumcenter** of the triangle.

The circle is **Circumsribed about the Triangle**.

8. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC to an obtuse triangle.

Where does the Circumcenter lie? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

9. Right-Click on one of the perpendicular bisectors and choose Hide Perpendicular Line.

10. Create segments$ \overbar{GA}$ , $\overbar{GB} $, and $ \overbar{GC}$. Find the length of each segment.

11. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice about the distance from the Circumcenter to each of the vertices of ∆ABC?

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**III. Incenter of a Triangle**

Open a new Geometer’s Sketchpad Document

1. Construct and label an acute scalene ∆ABC. Right-click to make the segments thick.

2. We want to create the angle bisector for each angle of the triangle.

- Click and Highlight Point B, then Point A, then Point C. (The vertex for the angle you want to measure must be clicked 2nd.)
- From the **Construct** menu, select Angle Bisector.
- Repeat for  and 

What do you notice about the intersection of the three angle bisectors?

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3. Place Point D at the point of concurrency.

4. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

For what type of triangle, if any, will the point of concurrency lie outside of ∆ABC?

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The point of concurrency of the Angle Bisectors of a triangle is called the
**Incenter** of the triangle.

The circle is **Inscribed inside the Triangle**.

5. Hide each angle bisector ray.

6. Construct a Perpendicular Line through Point D to $ \overbar{AB}$. Label the intersection
Point E. Construct a Perpendicular Line through Point D to $ \overbar{BC}$. Label the intersection
Point F. Construct a Perpendicular Line through Point D to $ \overbar{AC}$. Label the intersection
Point G.

7. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC. (Notice these lines are NOT necessarily bisectors.)

8. Hide each perpendicular line.

9. Create segments$ \overbar{DE}$ , $\overbar{DF} $, and $ \overbar{DG}$. Find the length of each segment.

10. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What do you notice about the distance from the Incenter to each of the vertices of ∆ABC?

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11. Click on the **Circle Tool**. Click on Point D and extend the edge of the circle to Point E.

12. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

How would you describe the circle that you constructed?

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**IV. Centroid of a Triangle**

Open a new Geometer’s Sketchpad Document

1. Construct and label an acute scalene ∆ABC. Right-click to make the segments thick.

2. Construct the midpoint D of $\overbar{AB}$, midpoint E of $\overbar{AC}$, and midpoint F of $\overbar{BC}$.

3. Create medians $\overbar{CD}$, $\overbar{BE},$ and $\overbar{AF}$.

4. Place Point G at the intersection of the medians.

The point of concurrency of the Medians of a triangle is called the
**Centroid** of the triangle**.**

5. Find the Length of $\overbar{AG},$ and $\overbar{GF}$.

6. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

What mathematical relationship do you see between the two segments that make up median $\overbar{AF}$?

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7. Delete the two distance calculations and repeat process with medians $\overbar{CD}$ and $\overbar{BE},$ to verify the above finding.

The longest of the two segments contained within the median connects the…

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**V. Orthocenter of a Triangle**

Open a new Geometer’s Sketchpad Document

1. Construct and label an acute scalene ∆ABC. Right-click to make the segments thick.

2. Construct the altitudes of ∆ABC by constructing a Perpendicular Line through Point A to $ \overbar{BC}$. Label the intersection Point D. Construct a Perpendicular Line through Point B to $ \overbar{AC}$. Label the intersection Point E. Construct a Perpendicular Line through Point C to $ \overbar{AB}$. Label the intersection Point F.

3. Place Point G at the intersection of the altitudes.

The point of concurrency of the Altitudes of a triangle is called the
**Orthocenter** of the triangle**.**

4. Using the Pointer tool, click and Hold point C. Drag your cursor around to change the size and shape of ∆ABC.

Does the Orthocenter ever lie on one of the vertices of the triangle? If so, what kind of triangle would this be?

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Does the Orthocenter ever lie outside of the triangle? If so, what kind of triangle would this be?

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