Block 6

Coordinate Proofs

Coordinate proofs are used to prove quadrilaterals or properties of quadrilaterals using the coordinate plane.

To Prove:	You Must:
Two Sides are Congruent	Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Two Segments Bisect Each Other	Midpoint Formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
Two Sides are Parallel or Perpendicular	Slope Formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$ Parallel $\rightarrow$ same

Perpendicular $\rightarrow$ neg. recip slopes

Proving Quadrilaterals By Definition

Parallelogram:

A Quad with both pairs of opposite sides parallel

4 Slope Formulas : opp sides need same slope

Rectangle:

A Quad with 4 right angles

4 slope Formulas : 4 pairs slope of of adj sides are neg. recip.

Rhombus:

A Quad with 4 congruent sides

4 distance Formulas:

Square:

A Quad with 4 right angles and 4 congruent sides

4 Slope and 4 distances see rhombus and rectange

Trapezoid:

A Quad with one pair of parallel sides and one pair of non-parallel sides

4 Slopes 2 opp are same 2 opp are not same

Isosceles Trapezoid:

A Quad with one pair of parallel sides and one pair of congruent non-parallel sides

4 slopes 2 distance Formulas

Kite:

A Quad with two pairs of adjacent sides congruent and no opposite sides congruent

4 distance Formulas : 2 pairs adjacent $\cong$

no opp sides are $\cong$

Proving Quadrilaterals With Cycles

To Prove a Quadrilateral is a **Parallelogram**
Show <u>ONE</u> of the following:

1. BOTH pairs of opposite sides congruent.

4 distance formulas

2. BOTH pairs of opposite sides parallel.

4 slope formulas

* 3. ONE pair of opposite sides congruent AND parallel.

2 distance and 2 slopes (same pair of opp sides)

S 4. Diagonals bisect each other.

2 Midpoints of Diag. (Same midpt)

To Prove a Quadrilateral is a **Rhombus**
First, prove it's a Parallelogram
Then, show <u>ONE</u> of the following:

adjacent
1. ONE pair of consecutive sides congruent.

2 distance formulas

S 2. Diagonals are perpendicular.

2 slopes (neg. recip.)

To Prove a Quadrilateral is a **Rectangle**
First, prove it's a Parallelogram
Then, show <u>ONE</u> of the following:

S 1. Diagonals congruent.

2 distence formulas

2. One right angle.

~~S~~ 2 slopes formula (neg. recip)

To Prove a Quadrilateral is a **Square**
First, prove it's a Parallelogram
Then, prove it's a Rhombus
Then, prove it's a Rectangle.

Proving Quadrilaterals with Cycles

Non-Parallelograms

To Prove a Quadrilateral is a **Trapezoid**
Show BOTH of the following:

1. ONE pair of opposite sides parallel.

   2 slopes (same slope)

2. ONE pair of opposite sides non-parallel.

   2 slopes (not same slopes)

To Prove a Quadrilateral is an **Isosceles Trapezoid**
First, show it's a trapezoid
Then, show ONE of the following:

1. NON-PARALLEL sides are congruent.

   2 distance formula

2. Diagonals are congruent.

   2 distance formulas

To Prove a Quadrilateral is a **Kite**
Always Prove by Definition
Show BOTH

1. 2 pairs of adjacent sides congruent.

   4 distances

2. No opposite sides congruent.

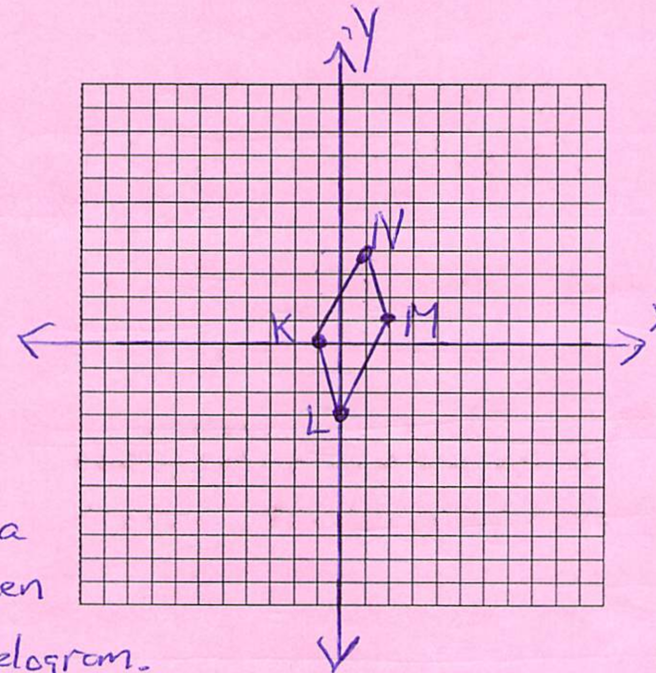
   Show using above

Coordinate Proofs Practice

24) The vertices of quadrilateral *KLMN* are *K*(-1,0), *L*(0,-3), *M*(2,1) and *N*(1,4). Show that *KLMN* is a parallelogram.

$m_{\overline{KL}} = \frac{-3}{1} = -3$ $\quad m_{\overline{KN}} = \frac{4}{2} = 2$

$m_{\overline{NM}} = \frac{-3}{1} = -3$ $\quad m_{\overline{LM}} = \frac{4}{2} = 2$



$\overline{KL} \parallel \overline{NM}$ and $\overline{KN} \parallel \overline{LM}$

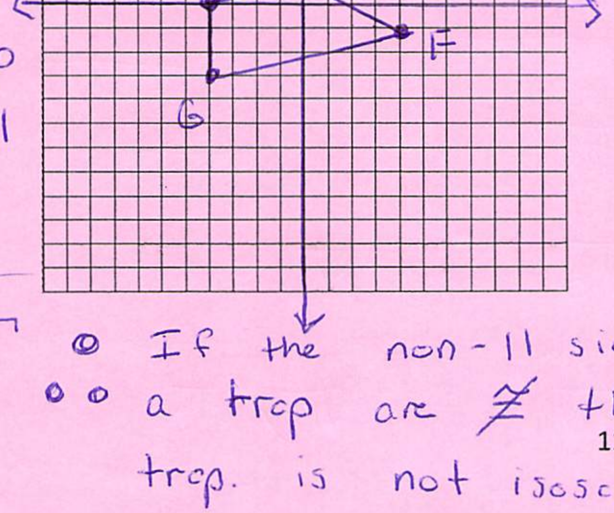
because they have = slopes

- If opposite sides of a quad are parallel then the quad is a parallelogram.

25) Quadrilateral *DEFG* has vertices *D*(-4,0), *E*(0,1), *F*(4,-1), and *G*(-4,-3). Show that *DEFG* is a trapezoid and not an isosceles trapezoid.

$m_{\overline{DE}} = \frac{1}{4}$ $\quad m_{\overline{GF}} = \frac{2}{8} = \frac{1}{4}$ $\quad m_{\overline{DG}} = \text{undefined}$

$m_{\overline{EF}} = \frac{-2}{4} = -\frac{1}{2}$



$\overline{DE} \parallel \overline{GF}$ → same slope

$\overline{DG} \nparallel \overline{EF}$ → not same slope

- If a quad has 1 pair opp sides ∥ and the other pair not ∥ then the quad is a ~~par~~ trap.

$d_{\overline{DG}} = 3$ $\quad d_{\overline{EF}} = \sqrt{(4-0)^2 + (-1-1)^2}$

$= \sqrt{(4)^2 + (-2)^2}$

$= \sqrt{20}$

$\overline{DG} \ncong \overline{EF}$

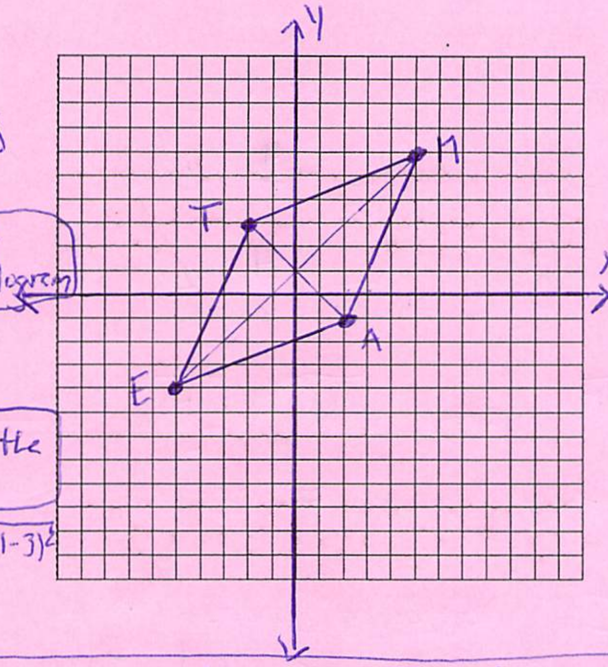
- If the non-∥ sides of a trap are $\ncong$ then the trap. is not isosceles.

26) Jim is experimenting with a new drawing program on his computer. He created quadrilateral *TEAM* with coordinates *T*(-2,3), *E*(-5,-4), *A*(2,-1), and *M*(5,6). Jim believes that he has created a rhombus but not a square. Prove that Jim is correct.

$M_{\overline{EM}} = (0,1)$ $\quad$ $M_{\overline{AT}} = (0,1)$

If midpts are the same for the diag then they bisect each other

$\therefore$ If a quad has diags. that bisect each other the quad is a parallelogram



$m_{\overline{EM}} = 1$ $\quad$ $m_{\overline{AT}} = -1$ $\quad$ $\overline{EM} \perp \overline{AT}$

$\therefore$ If a parallelogram has $\perp$ diags then the parallelogram is a rhombus

$d_{\overline{EM}} = \sqrt{(5+5)^2 + (6+4)^2}$ $\quad$ $d_{\overline{AT}} = \sqrt{(2+2)^2 + (-1-3)^2}$

$= \sqrt{200}$ $\quad$ $\overline{AT} \ncong \overline{EM}$ $\quad$ $= \sqrt{32}$

$\therefore$ If rhombus has non-$\cong$ diags. then the rhombus is not a square.

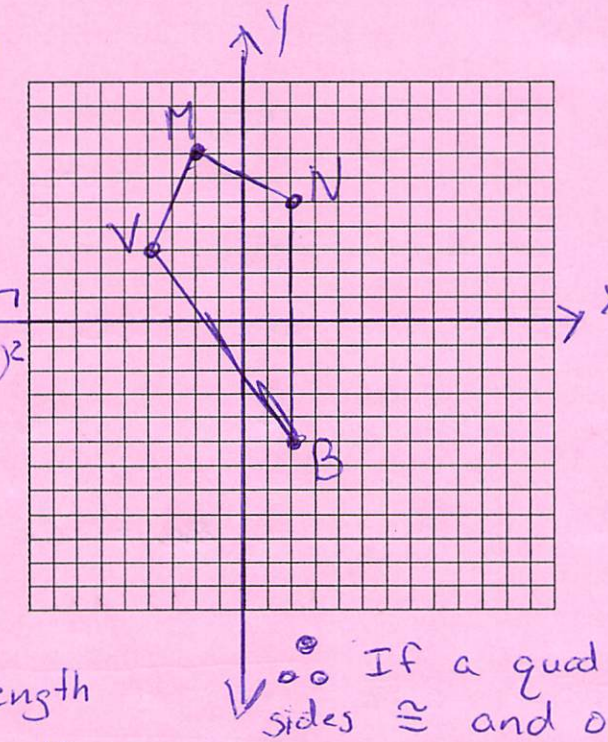
27) Quadrilateral *MNBV* has coordinates *M*(-2, 7), *N*(2, 5), *B*(2, -5), and *V*(-4, 3). Prove that *MNBV* is a kite.

$d_{\overline{NB}} = 10$

$d_{\overline{VB}} = \sqrt{(-4-2)^2 + (-5-3)^2}$

$= \sqrt{(-6)^2 + (-8)^2}$

$= \sqrt{100} = 10$



$d_{\overline{MV}} = \sqrt{(-2+4)^2 + (7-3)^2}$ $\quad$ $d_{\overline{MN}} = \sqrt{(-2-2)^2 + (7-5)^2}$

$= \sqrt{(2)^2 + (4)^2}$ $\quad$ $= \sqrt{(-4)^2 + (2)^2}$

$= \sqrt{20}$ $\quad$ $= \sqrt{20}$

$\overline{NB} \cong \overline{VB}$ and $\overline{MV} \cong \overline{MN}$ because they have the same length

$\overline{NB} \ncong \overline{MV}$ and $\overline{VB} \ncong \overline{MN}$ because they do not have the same length.

$\therefore$ If a quad has adj ~~opp~~ sides $\cong$ and opp sides not $\cong$ then the quad is a kite.

28) Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram *ABCD* are *A*(0,0), *B*(5,2), and *C*(6,5). Find the coordinates of point *D* and sketch parallelogram *ABCD* on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.

$m_{\overline{AB}} = m_{\overline{DC}}$ $\qquad$ $m_{\overline{BC}} = m_{\overline{AD}}$

$\frac{2}{5} = \frac{y-5}{x-6}$ $\qquad$ $\frac{3}{1} = \frac{y}{x}$

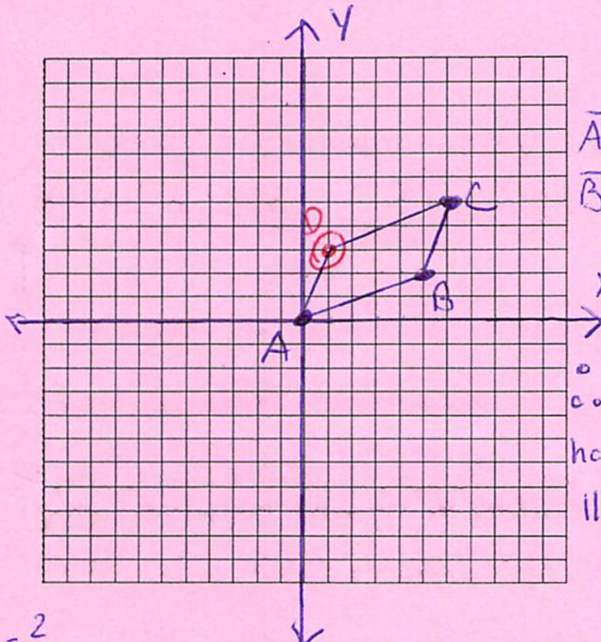
$y = 3x$

$\frac{2}{5} = \frac{3x-5}{x-6}$

$2x - 12 = 15x - 25$

$-13x = -13$

$\boxed{x=1}$ $\qquad$ $\boxed{y=3}$ $\qquad$ $(1,3)$



$\overline{AB} \parallel \overline{DC}$ } Same slope
$\overline{BC} \parallel \overline{AD}$ } Slope

∴ If a quad has opp sides ∥ then it is a parallelogram

$m_{\overline{AB}} = \frac{2}{5}$ $\qquad$ $m_{\overline{DC}} = \frac{2}{5}$ $\qquad$ $m_{\overline{BC}} = 3$ $\qquad$ $m_{\overline{AD}} = 3$

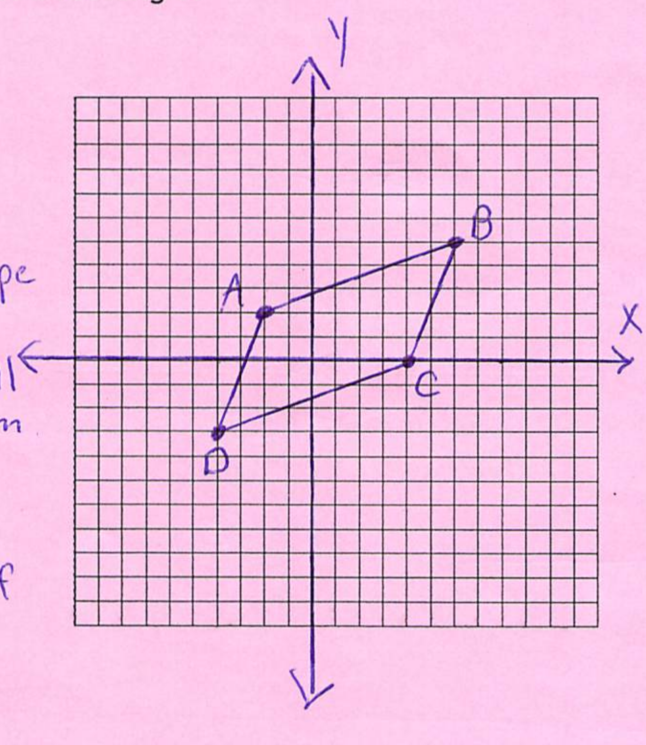
29) Given: *A*(-2,2), *B*(6,5), *C*(4,0), *D*(-4,-3)
Prove: *ABCD* is a parallelogram but not a rectangle.

$m_{\overline{AB}} = \frac{3}{8}$ $\qquad$ $m_{\overline{AD}} = \frac{5}{2}$

$m_{\overline{DC}} = \frac{3}{8}$ $\qquad$ $m_{\overline{BC}} = \frac{5}{2}$

$\overline{AB} \parallel \overline{DC}$ $\qquad$ $\overline{AD} \parallel \overline{BC}$] Same slope

∴ If 2 pair of opp sides are ∥ then the quad is a parallelogram



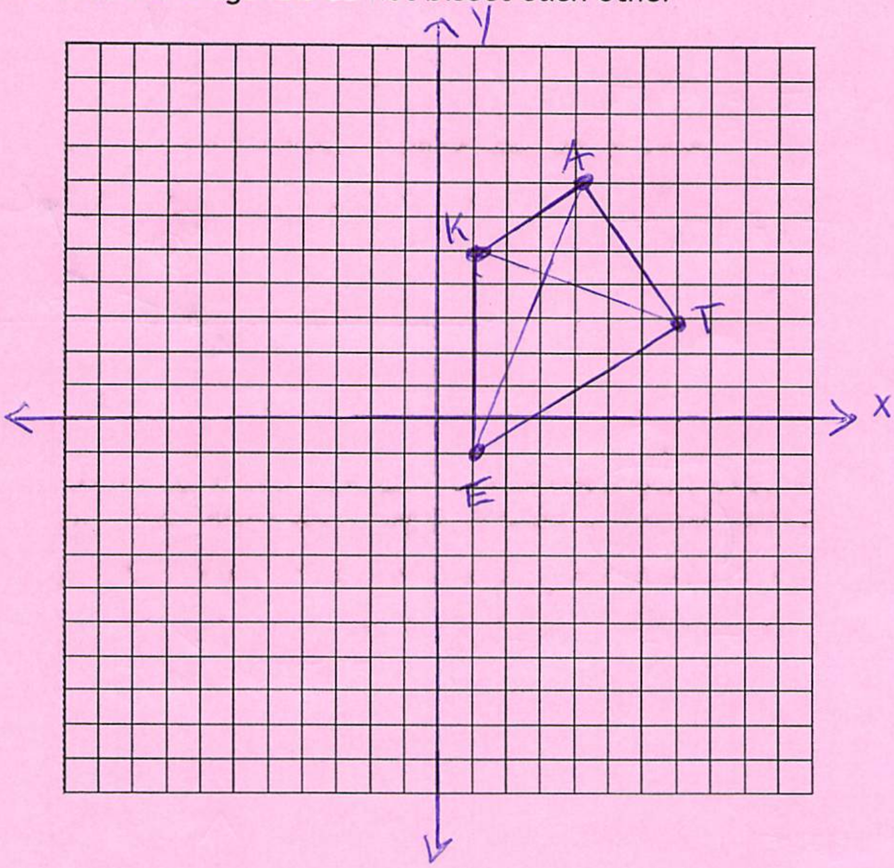
$\overline{AB} \not\perp \overline{AD}$ because slopes are not negative recips of each other.

∴ $\angle A \neq 90°$

∴ If a parallelogram has an angle that is not a right ∠ then the parallelogram is not a rectangle.

30) Quadrilateral *KATE* has vertices *K*(1,5), *A*(4,7), *T*(7,3), and *E*(1,-1).

a. Prove that *KATE* is a trapezoid.
b. Prove that *KATE* is *not* an isosceles trapezoid.
c. Prove that the diagonals do not bisect each other



a)

$m_{\overline{KA}} = \frac{2}{3}$ $m_{\overline{ET}} = \frac{2}{3}$ $m_{\overline{KE}}$ = undefined $m_{\overline{AT}} = \frac{-4}{3}$

$\overline{KA} \parallel \overline{ET}$ → ~~same slope~~ $\overline{KE} \nparallel \overline{AT}$] different slopes

∴ If a quad has a pair of ∥ and a pair of non-∥ sides then the quad is a trap.

$d_{\overline{KE}} = 6$

b)

$d_{\overline{AT}} = \sqrt{(7-4)^2+(3-7)^2}$
$= \sqrt{25} = 5$

∴ If a trap has non-∥ sides not ≅ then the trap is not isos.

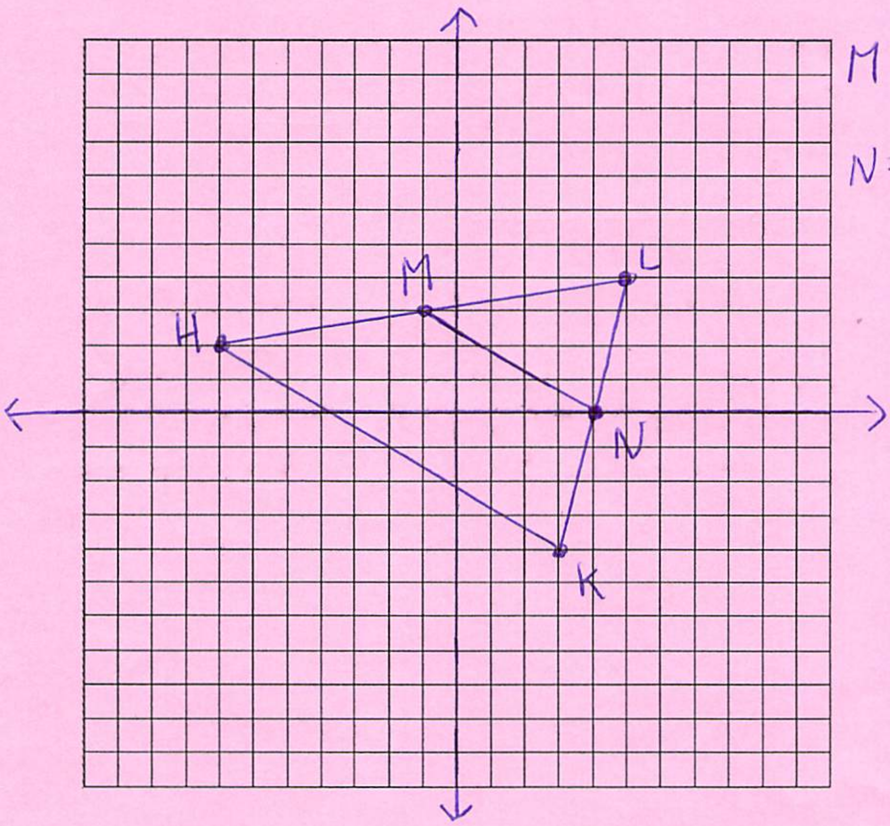
c) $M_{\overline{KT}} = (4,4)$
$M_{\overline{AE}} = (\frac{5}{2}, 3)$] don't have same midpt so they don't bisect each other.

20

31) Triangle *HKL* has vertices *H*(-7, 2), *K*(3, -4), and *L*(5, 4). The midpoint of $\overline{HL}$ is *M* and the midpoint of $\overline{KL}$ is *N*.

a. Determine and state the coordinates of point *M* and *N*.

b. ~~Prove~~ Verify that $\overline{MN}$ ~~is a midsegment.~~ has the properties of a midsegment.



$M = (-1, 3)$

$N = (4, 0)$

$m_{\overline{HK}} = \frac{-6}{10} = \frac{-3}{5}$ $\qquad m_{\overline{MN}} = \frac{-3}{5}$ $\qquad \boxed{\overline{HK} \parallel \overline{MN}}$

$d_{\overline{HK}} = \sqrt{(-7-3)^2 + (2+4)^2}$

$= \sqrt{(-10)^2 + (6)^2}$

$= \sqrt{136} = 11.66$

$d_{MN} = \sqrt{(4+1)^2 + (3-0)^2}$

$= \sqrt{(5)^2 + (3)^2}$

$= \sqrt{34} = 5.83$ $\qquad \boxed{\frac{1}{2}\overline{HK} = \overline{MN}}$

- Midsegment $\overline{MN}$ is parallel and half the length of $\overline{HK}$. It satisfies the properties of a midsegment.