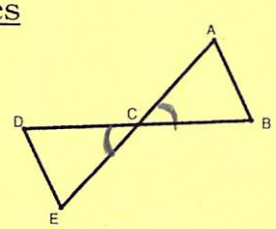
Proving Triangles Congruent: Common Cycles

The key to using the givens is knowing your vocabulary. Our goal is to find corresponding parts (sides or angles) of triangles that we can prove congruent. Your job is to know which vocabulary leads to which congruent parts.

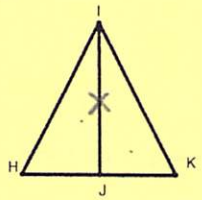
In addition to the givens there are three items that you can introduce into a proof from the picture and do not require any prior information. These are our FREEBIES!

Vertical Angles



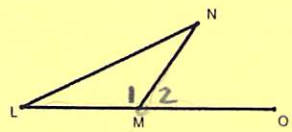
Statements	Reasons
(1) $\angle DCE \cong \angle ACB$	(1) Vertical $\angle$'s are $\cong$

Reflexive Property



Statements	Reasons
(1) $\overline{IJ} \cong \overline{IJ}$	(1) Reflexive Prop

Linear Pair



Statements	Reasons
(1) $\angle 1$ and $\angle 2$ form linear pair	(1) If 2 lines intersect then adjacent $\angle$'s form a linear pair
(2) $\angle 1 + \angle 2 = 180$	(2) If 2 $\angle$'s form a linear pair then they are supplementary

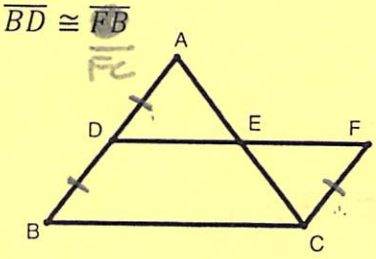
Using the givens and the FREEBIES in combination leads to sets of steps that occur together in many different proofs. These are called *cycles*. Learning these cycles will greatly assist you when working through proofs.

- Transitive Property
- Congruent Supplements
- Congruent Complements
- Segment Addition/Subtraction
- Angle Addition/Subtraction
- Congruent Segment/Angle Bisectors

Transitive Property

Given: $\overline{BD} \cong \overline{AD}$, $\overline{FB} \cong \overline{AD}$ (FC)

Prove: $\overline{BD} \cong \overline{FB}$ (FC)

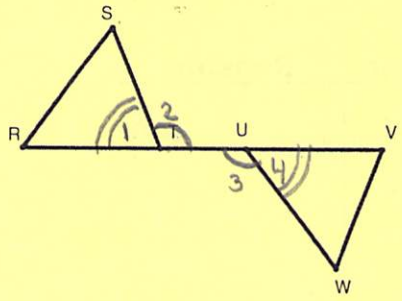


Statements	Reasons
① $\overline{BD} \cong \overline{AD}$, $\overline{FC} \cong \overline{AD}$	① Given
② $\overline{BD} \cong \overline{FC}$	② Transitive Property

Congruent Supplements

Given: $\angle STU \cong \angle WUT$

Prove: $\angle STR \cong \angle WUV$

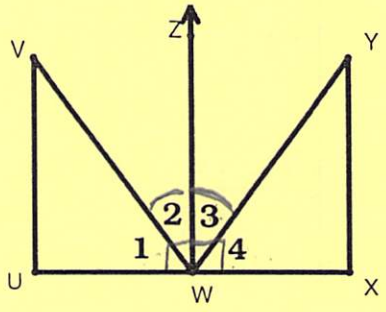


Statements	Reasons
① $\angle STU \cong \angle WUT$	① Given
② $\angle 1$ and $\angle 2$ are supp. $\angle 3$ and $\angle 4$ are supp	② Linear pairs of $\angle$'s are supplementary
③ $\angle STR \cong \angle WUV$	③ If 2 $\angle$'s are supplements of $\cong$ $\angle$'s then the $\angle$'s are $\cong$

Congruent Complements

Given: $\overline{WZ} \perp \overline{UX}$ $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 4$

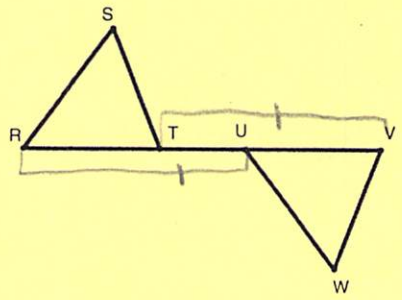


Statements	Reasons
① $\overrightarrow{WZ} \perp \overline{UX}$ $\angle 2 \cong \angle 3$	① Given
② $\angle ZWU \cong \angle ZWX$	② If 2 seg are $\perp$ then 2 $\cong$ 90° $\angle$'s formed
③ $\angle 1 \cong \angle 4$	③ If 2 $\angle$'s are Complements of $\cong$ $\angle$'s then the $\angle$'s are $\cong$

Segment Addition/Subtraction

Given: $\overline{RU} \cong \overline{VT}$

Prove: $\overline{RT} \cong \overline{VU}$

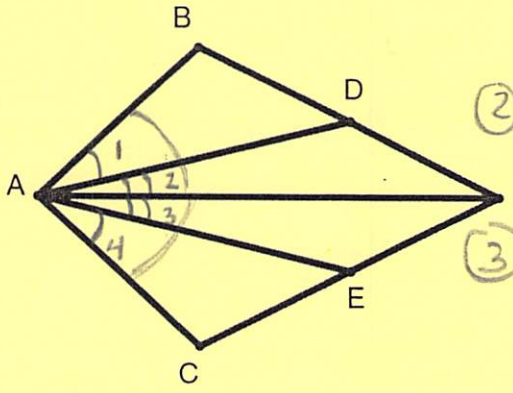


Statements	Reasons
1) $\overline{RU} \cong \overline{VT}$	1) Given
2) $\overline{TU} \cong \overline{TU}$	2) Reflexive Prop.
3) $\overline{RU} - \overline{TU} \cong \overline{VT} - \overline{TU}$	3) Subtraction Prop. of Equality
4) $\overline{RT} \cong \overline{VU}$	4) Segment Subtraction Postulate

Angle Addition/Subtraction

Given: $\angle BAD \cong \angle CAE$
$\angle DAF \cong \angle EAF$

Prove: $\angle BAF \cong \angle CAF$

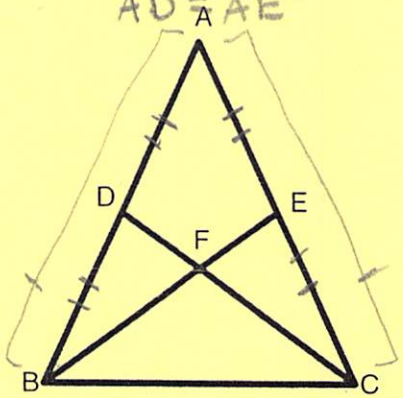


Statements	Reasons
1) $\angle BAD \cong \angle CAE$, $\angle DAF \cong \angle EAF$	1) Given
2) $\angle 1 + \angle 2 \cong \angle 3 + \angle 4$	2) Addition prop. of equality
3) $\angle BAF \cong \angle CAF$	3) Angle Addition Postulate

Congruent Segment/Angle Bisectors

Given: $\overline{AB} \cong \overline{AC}$, $\overline{CD}$ bisects $\overline{AB}$, $\overline{BE}$ bisects $\overline{AC}$

Prove: ~~$\angle BAF \cong \angle CAF$~~ $\overline{AD} \cong \overline{AE}$



Statements	Reasons
1) $\overline{AB} \cong \overline{AC}$, $\overline{CD}$ bisects $\overline{AB}$, $\overline{BE}$ bisects $\overline{AC}$	1) Given
2) $\overline{AD} \cong \overline{BD}$, $\overline{AE} \cong \overline{CE}$	2) If a segment bisects another seg. 2 $\cong$ seg are formed.
3) $\overline{AD} \cong \overline{AE}$	3) If 2 $\cong$ seg are bisected then all seg. formed are $\cong$