17. A tree is located 30 feet east of a fence that runs north to south. Kelly tells her brother Billy that their dog buried Billy's hat a distance of 15 feet from the fence and also 20 feet from the tree. Draw a sketch to show where Billy should dig to find his hat. Based on your sketch, how many locations for the hat are possible?

18. Point P is x inches from line ℓ. If there are exactly three points that are 2 inches from line ℓ and also 6 inches from P, what is the value of x?

19. Point P is located on $\overleftrightarrow{AB}$. How many points are 3 units from $\overleftrightarrow{AB}$ and 5 units from point P? Draw a diagram to support your answer.

20. Lines AB and CD are parallel to each other and 6 inches apart. Point P is located between the two parallel lines and 1 inch from $\overleftrightarrow{AB}$. How many points are equidistant from $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and, at the same time, 2 inches from point P? Draw a diagram to support your answer.

21. a. Describe completely the locus of points 2 units from the line whose equation is $x = 3$.
    b. Describe completely the locus of points n units from the point $P(3,2)$.
    c. Determine the total number of points that satisfy the locus conditions in parts a and b simultaneously for $n = 2$?

22. Point M is the midpoint of $\overline{AB}$.
    a. Describe fully the locus of all points in a plane that are
       (1) equidistant from A and B
       (2) 6 units from $\overleftrightarrow{AB}$
       (3) d units from M
    b. For what value of d will there be exactly two points that simultaneously satisfy all three conditions in part **a**.

23. Point P is on $\overleftrightarrow{AB}$.
    a. Describe fully the locus of points:
       (1) d units from $\overleftrightarrow{AB}$
       (2) D units from P.
    b. Find the number of points that satisfy the two locus conditions when
       (1) $D = d$
       (2) $D < d$
       (3) $D > d$

9.3 CONCURRENCY THEOREMS

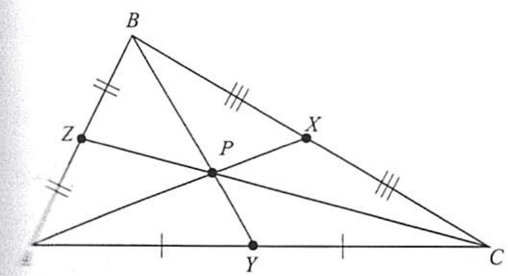
KEY IDEAS

Concurrent lines are three or more lines that intersect in the same point. The mutual point of intersection is called the **point of concurrency**. In a triangle, the following sets of lines are concurrent:

- The three medians
- The three altitudes
- The perpendicular bisectors of the three sides
- The three angle bisectors

Concurrency of Medians

The three medians of a triangle are concurrent in a point called the **centroid** of the triangle. The distance from each vertex to the centroid is two-thirds of the length of the entire median drawn from that vertex, as shown in Figure 9.1. Thus, if $AX = 9$, then $AP = \frac{2}{3} \times 9 = 6$ and $PX = 3$.



Points X, Y, and Z are midpoints:

- $AP = \frac{2}{3}AX$
- $BP = \frac{2}{3}BY$
- $CP = \frac{2}{3}CZ$

Figure 9.1 Point P is the *centroid* of the triangle.

The centroid of a triangle is its *center of gravity*. A metal triangular plate can be made to balance horizontally in space by placing a single support directly underneath the centroid of the triangle.

Concurrency of Perpendicular Bisectors

The perpendicular bisectors of the three sides of a triangle are concurrent in a point equidistant from the vertices of the triangle. In Figure 9.2, the perpendicular bisectors of the sides of $\triangle ABC$ are concurrent in point O. Since point O is equidistant from the vertices of the triangle, $OA = OB = OC$.

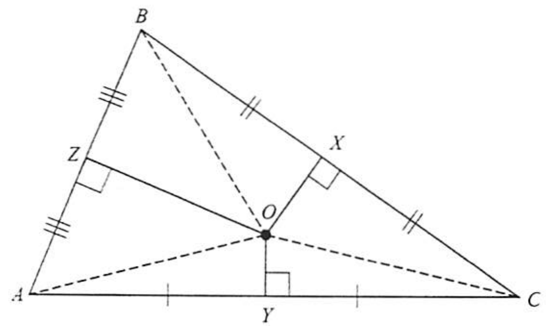
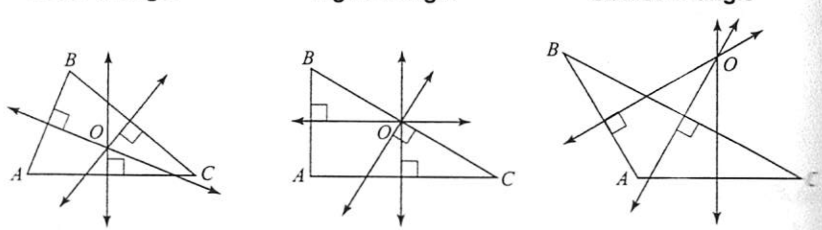


Figure 9.2 Point O is the circumcenter of $\triangle ABC$.

Using OA as a radius and point O as a center, a circle can be *circumscribed* about $\triangle ABC$ so that each of its vertices are points on the circle. Point O is called the **circumcenter** of the triangle.

Location of the Circumcenter of a Triangle

The circumcenter of a triangle can fall in the interior of the triangle, on a side of the triangle, or in the exterior of the triangle, as illustrated in Figure 9.3.



Point O in the interior of the acute triangle.

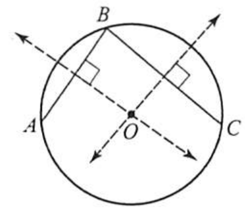
Point O on the hypotenuse of a right triangle.

Point O in the exterior of the obtuse triangle.

Figure 9.3 Locating the circumcenter O of a triangle.

MATH FACTS

The center of a circle can be located by finding the point of intersection of the perpendicular bisectors of any two non-parallel chords of the circle.



Concurrency of Angle Bisectors

The three angle bisectors of a triangle are concurrent in a point equidistant from the sides of the triangle. In Figure 9.4, the bisectors of the angles of $\triangle ABC$ are concurrent in point Q. Since point Q is equidistant from the sides of the triangle, $QX = QY = QZ$.

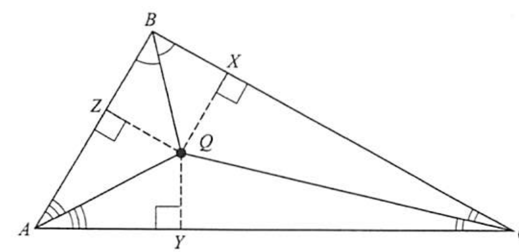


Figure 9.4 The bisectors of the angles are concurrent at a point Q equidistant from the sides of $\triangle ABC$.

Using QX as the radius and point Q as the center, a circle can be *inscribed* in $\triangle ABC$ so that points X, Y, and Z are on the circle, as shown in Figure 9.5. Point Q is called the **incenter** of the triangle. Unlike the circumcenter of a triangle, the incenter of a triangle always lies in the interior of the triangle.

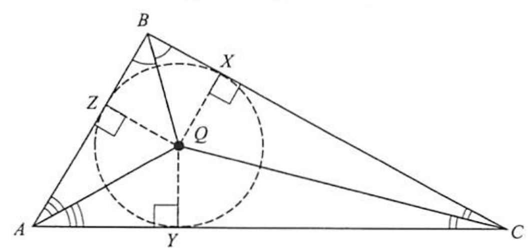


Figure 9.5 Point Q is the incenter of $\triangle ABC$.

Concurrency of Altitudes

The altitudes of a triangle, extended if necessary, are concurrent in a point called the **orthocenter** of the triangle. The orthocenter of a triangle can fall in the interior of the triangle, on a side of the triangle, or in the exterior of the triangle, as illustrated in Figure 9.6.

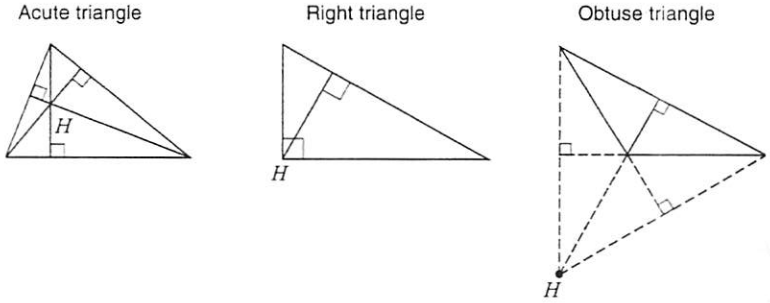


Figure 9.6 Point *H* is the *orthocenter* of each triangle.

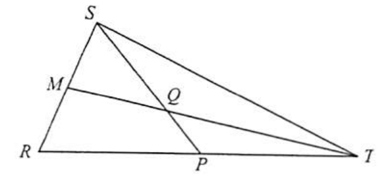
- In an acute triangle, the orthocenter lies in the interior of the triangle.
- In a right triangle, either leg serves as the altitude drawn to the other leg. The point of concurrency of the three altitudes is the vertex of the right angle.
- In an obtuse triangle, the altitudes and a side need to be extended so that the orthocenter falls in the exterior of the triangle.

MATH FACTS

Set of Lines in a Triangle	Point of Concurrency	Related Facts
Medians	Centroid	Centroid divides each median into segment lengths with the ratio 2:1 as measured from each vertex.
Perpendicular bisectors of sides	Circumcenter	Circumcenter is the center of the circumscribed circle. In a right triangle, it lies on the hypotenuse.
Angle bisectors	Incenter	Incenter is the center of the inscribed circle and always lies in the interior of the triangle.
Altitudes	Orthocenter	In a right triangle, the orthocenter is at the vertex of the right angle.

Check Your Understanding of Section 9.3

A. *Multiple Choice.*



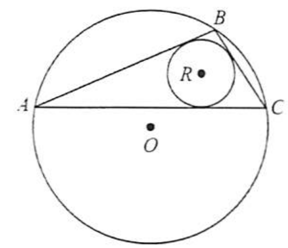
1. In $\triangle RST$, medians $\overline{TM}$ and $\overline{SP}$ are concurrent at point Q. If $TQ = 3x - 1$ and $QM = x + 1$, what is the length of median $\overline{TM}$?
   (1) 3 (2) 8 (3) 12 (4) 11

2. If circle O is circumscribed about $\triangle ABC$, then point O always lies on
   (1) side $\overline{AC}$
   (2) the median to side $\overline{AC}$
   (3) the bisector of $\angle ABC$
   (4) the perpendicular bisector of $\overline{AC}$

3. In $\triangle ABC$, points J, K, and L are the midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively. If the three medians of the triangle intersect at point P and the length of $\overline{LP}$ is 6, what is the length of median $\overline{BL}$?
   (1) 18 (2) 12 (3) 9 (4) 4

4. If circle O is inscribed in $\triangle ABC$, then point O is always on
   (1) side $\overline{AC}$
   (2) the median to side $\overline{AC}$
   (3) the bisector of $\angle ABC$
   (4) the perpendicular bisector of $\overline{AB}$

5. Given right triangle ABC with right angle at C. The locus of points equidistant from points A and B intersects $\overline{AB}$ at the
   (1) center of the inscribed circle
   (2) center of the circumscribed circle
   (3) orthocenter of the triangle
   (4) centroid of the triangle



6. In the accompanying figure, O is the center of the circle circumscribed about scalene $\triangle ABC$ and R is the center of the circle inscribed in scalene $\triangle ABC$.
Which statement is *false*?
(1) Point R is equidistant from $\overline{AC}$ and $\overline{BC}$.
(2) Point O is equidistant from points B and C.
(3) Point R lies on the bisectors of angles A and C.
(4) Point O is the point at which the altitudes drawn to sides $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$ intersect.

B. *Show or explain how you arrived at your answer.*

7. Given right triangle ABC with right angle at C.
a. Describe the locus of points equidistant from sides $\overline{AC}$ and $\overline{AB}$.
b. In how many points, if any, will the locus described in part a intersect the locus of points equidistant from sides $\overline{AB}$ and $\overline{BC}$.
c. The locus described in parts a and b can be used to determine the
(1) center of the inscribed circle
(2) center of the circumscribed circle
(3) orthocenter of the triangle
(4) centroid of the triangle

9.4 WRITING AN EQUATION OF A LINE

KEY IDEAS

The set of all ordered pairs (x, y) that satisfy a two-variable linear equation is represented by a line in the coordinate plane. To be able to write an equation of an oblique (slanted) line, you need to know two facts about the line:

- The slope and the coordinates of a point on the line or
- The coordinates of two points on the line.

Slope–Intercept Equation: $y = mx + b$

When a two-variable linear equation is solved for y, it has the general form $y = mx + b$, where

- m, the coefficient of x, is the slope of the line
- b, the y-intercept, indicates where the line crosses the y-axis

To read the slope and y-intercept of a line from its equation, it may be necessary to rewrite it in $y = mx + b$ form. If an equation of line ℓ is $2y - 6x = 10$, then $\frac{2y}{2} = \frac{6x}{2} + \frac{10}{2}$ and $y = 3x + 5$ so the slope of line ℓ is 3 and its y-intercept is 5.

Example 1

An equation of line ℓ is $2y = 3x + 6$. Which equation represents a line that is perpendicular to line ℓ?

(1) $y = \frac{3}{2}x + 2$
(2) $y = -\frac{3}{2}x + 2$
(3) $y = \frac{2}{3}x - 3$
(4) $y = -\frac{2}{3}x - 3$

Solution: Perpendicular lines have slopes that are negative reciprocals. Compare the slope of line ℓ with the slopes of the lines in the answer choices.

- Find the slope of line ℓ. If $2y + 3x = 6$, then solving for y gives $y = -\frac{3}{2}x + 3$.
  Hence, the slope of line ℓ is $-\frac{3}{2}$.
- The slope of the required line must be $\frac{2}{3}$; the negative reciprocal, $-\frac{3}{2}$.
- Because the slope of the line in answer choice (3) is $\frac{2}{3}$, this line is perpendicular to line ℓ.

The correct choice is **(3)**.

Example 2

The equation of line r is $3y + 6x = 12$ and the equation of line s is $8y - 12 = 4x$. Which statement is true about lines r and s?

(1) $r \parallel s$
(2) $r \perp s$
(3) lines r and s coincide
(4) lines r and s have the same y-intercept

Solution: Rewrite each equation in slope-intercept form.

- Line r: $3y + 6x = 12$, so $3y = -6x + 12$ and $y = -2x + 4$.
- Line s: $8y - 12 = 4x$, so $8y = 4x + 12$ and $y = \frac{1}{2}x + \frac{3}{2}$.
- The slope of line r is -2, and the slope of line s is $\frac{1}{2}$. Since -2 and $\frac{1}{2}$ are negative reciprocals, $r \perp s$.

The correct choice is **(2)**.

Example 3

Write an equation of the line that passes through the point $(-1, 3)$ and is parallel to the line whose equation is $y - 2x = 3$.

Solution: If $y - 2x = 3$, then $y = 2x + 3$ so the slope of this line is 2.

- Because parallel lines have the same slope, the slope of the required line is also 2, so its equation has the form, $y = 2x + b$.
- It is also given that $(-1, 3)$ is a point on the line. Find b by substituting $x = -1$ and $y = 3$ in $y = 2x + b$, which gives $3 = 2(-1) + b$, so $b = 5$.
- Since $m = 2$ and $b = 5$, an equation of the required line is $\mathbf{y = 2x + 5}$.

Point–Slope Equation: $y - k = m(x - h)$

In point–slope form, an equation of a line is written as $y - k = m(x - h)$, where (h,k) is any point on the line and m is the slope of the line. If the line that has a slope of -2 passes through the point $(4, 3)$, then its equation can be written in point–slope form by letting $h = 4$, $k = 3$, and $m = -2$:

$$y - \underbrace{k}_{\downarrow} = \underbrace{m}_{\downarrow}(x - \underbrace{h}_{\downarrow})$$

$$y - 3 = -2(x - 4)$$

If necessary, the equation $y - 3 = -2(x - 4)$ can be written in $y = mx + b$ form by removing the parentheses on the right side of the equation and isolating y:

$$\begin{aligned} y - 3 &= -2(x - 4) \\ y - 3 &= -2x + 8 \\ y &= -2x + 11 \end{aligned}$$

Example 4

Write an equation of the line ℓ that passes through $(6, -7)$ and is parallel to the line $y - 3x = -4$. What is the y-intercept of line ℓ?

Solution: Let m represent the slope of line ℓ and (h, k) represent the coordinates of any point on line ℓ.

- Rewrite $y - 3x = -4$ as $y = 3x - 4$. Because line ℓ is parallel to the given line, the slope of line ℓ is also 3.
- As it is also given that line ℓ passes through $(6, -7)$, $(h, k) = (6, -7)$.
- Write an equation of line ℓ using the point–slope form where $h = 6$, $k = -7$, and $m = 3$:

$$\begin{aligned} y - k &= m(x - h) \\ y - (-7) &= 3(x - 6) \\ y + 7 &= 3x - 18 \\ y &= 3x - 25 \end{aligned}$$

The y-intercept of line ℓ is **-25**.

Example 5

Find an equation of the line that is the perpendicular bisector of the line segment whose endpoints are $R(-8, 7)$ and $S(4, 3)$.

Solution:

- Find the midpoint $(\bar{x}, \bar{y})$ of $\overline{RS}$:

$$\bar{x} = \frac{-8+4}{2} = \frac{-4}{2} = -2 \quad \text{and} \quad \bar{y} = \frac{7+3}{2} = \frac{10}{2} = 5$$

The midpoint of $\overline{RS}$ is $(-2, 5)$.

- Find the slope of $\overline{RS}$:

$$m = \frac{\Delta y}{\Delta x} = \frac{3-7}{4-(-8)} = \frac{-4}{12} = -\frac{1}{3}$$

The slope of the perpendicular bisector of $\overline{RS}$ is the negative reciprocal of $-\frac{1}{3}$ or 3.

- Use the point–slope equation form where $(h, k) = (\bar{x}, \bar{y}) = (-2, 5)$ and $m = 3$:

$$\begin{aligned} y - k &= m(x - h) \\ y - 5 &= 3(x - (-2)) \\ \mathbf{y - 5} &= \mathbf{3(x + 2)} \end{aligned}$$

If required, the equation can also be written in $y = mx + b$ form:

$$\begin{aligned} y - 5 &= 3(x + 2) \\ y - 5 &= 3x + 6 \\ \mathbf{y} &= \mathbf{3x + 11} \end{aligned}$$

Example 6

The vertices of $\triangle ABC$ are $A(-3, -1)$, $B(1,7)$, and $C(6, -3)$.

a. Write an equation of the line that contains the median from C to $\overline{AB}$.
b. Write an equation of the line that contains the altitude from A to $\overline{BC}$.
c. Prove that the altitude determined in part b, when extended, passes through the point (7, 4).

Solution:

a. Label the point at which the median from C intersects side $\overline{AB}$ as point M.

- Find the midpoint, $M(\bar{x}, \bar{y})$, of $\overline{AB}$:

$$\bar{x} = \frac{-3+1}{2} = \frac{-2}{2} = -1 \qquad \text{and} \qquad \bar{y} = \frac{-1+7}{2} = \frac{6}{2} = 3$$

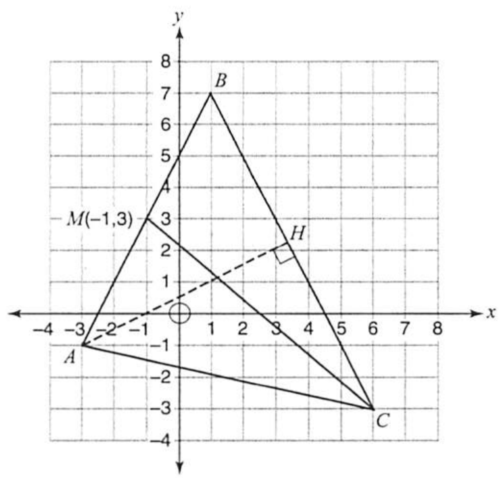
The midpoint is $M(-1, 3)$.

- Find the slope, m, of $\overline{CM}$:

$$m = \frac{\Delta y}{\Delta x} = \frac{3-(-3)}{-1-6} = -\frac{6}{7}$$

- Use the point–slope equation form where $(h, k) = M(-1, 3)$ and $m = -\frac{6}{7}$:

Equation of $\overline{CM}$: $\mathbf{y - 3 = -\frac{6}{7}(x - 3)}$



b. Label the point at which the altitude intersects $\overline{BC}$ as point H.

- Find the slope, m, of $\overline{BC}$:

$$m = \frac{\Delta y}{\Delta x} = \frac{-3-7}{6-1} = \frac{-10}{5} = -2$$

- Find the slope of $\overline{AH}$. Since $\overline{AH} \perp \overline{BC}$, slope of $\overline{AH} = \frac{1}{2}$.

- Use the point–slope equation form where $(h, k) = A(-3, -1)$ and $m = \frac{1}{2}$:

$$y - (-1) = \frac{1}{2}(x - (-3))$$

Equation of $\overline{AH}$: $\mathbf{y + 1 = \frac{1}{2}(x + 3)}$

If required, the equation can also be written in $y = mx + b$ form:

$$y + 1 = \frac{1}{2}(x + 3)$$

$$y + 1 = \frac{1}{2}x + \frac{3}{2}$$

Slope–intercept form of $\overline{AH}$: $y = \frac{1}{2}x + \frac{1}{2}$

c. Show that the point (7, 4) satisfies the equation of $\overline{AH}$:

$$y = \frac{1}{2}x + \frac{1}{2}$$

$$4 \;\Big|\; \frac{1}{2}(7) + \frac{1}{2}$$

$$\frac{7}{2} + \frac{1}{2}$$

$$\frac{8}{2}$$

$$4 = 4 \checkmark$$

Check Your Understanding of Section 9.4

A. Multiple Choice

1. Which line is perpendicular to the line whose equation is $5y + 6 = -3x$?
   (1) $y = -\frac{5}{3}x + 7$ (3) $y = -\frac{3}{5}x + 7$
   (2) $y = \frac{5}{3}x + 7$ (4) $y = \frac{3}{5}x + 7$

2. The graph of $x - 3y = 6$ is parallel to the graph of
   (1) $y = -3x + 7$ (3) $y = 3x - 8$
   (2) $y = -\frac{1}{3}x + 5$ (4) $y = \frac{1}{3}x + 8$

3. The graph of which equation is perpendicular to the graph of $y - 3 = \frac{1}{2}x$?
   (1) $y = -\frac{1}{2}x + 5$ (3) $y = 2x + 5$
   (2) $2y = x + 3$ (4) $y + 2x = 3$

4. Which is an equation of the line that is parallel to $y = 2x - 8$ and passes through the point (0,–3)?
   (1) $y = 2x + 3$ (3) $y = -\frac{1}{2}x + 3$
   (2) $y = 2x - 3$ (4) $y = -\frac{1}{2}x - 3$

5. Which is an equation of the line that is parallel to $y - 3x + 5 = 0$ and has the same y-intercept as $y = -2x + 7$?
   (1) $y = 3x - 2$ (3) $y = 3x + 7$
   (2) $y = -2x - 5$ (4) $y = -2x - 7$

B. Show or explain how you arrived at your answer.

6. a. Determine an equation of the line that is the perpendicular bisector of the line segment whose endpoints are $A(-1, 8)$ and $B(-5, 2)$.
   b. Determine the coordinates of the point at which the perpendicular bisector of $\overline{AB}$ intersects the y-axis.

7. Write an equation that describes the locus of points equidistant from the lines $y = 3x - 1$ and $y = 3x + 9$.

8. Given points $A(6, 3)$ and $B(2, 2)$.
   a. If A' is the image of point A after a reflection over the line $y = x$, find an equation of $\overleftrightarrow{A'B}$.
   b. Determine the number of square units in the area of $\triangle ABA'$.

9. Kim graphed the line represented by the equation $3y + 2x = 4$. Write an equation of a line that is
   a. Parallel to the line Kim graphed and that contains the point (1, –6).
   b. Perpendicular to the line Kim graphed and that passes through the origin.
   c. Neither parallel nor perpendicular to the line Kim graphed and that has the same y-intercept as the line Kim graphed.

10. The vertices of $\triangle RST$ are $R(2, 7)$, $S(8, 9)$, and $T(6, 3)$.
    a. Write an equation of the perpendicular bisector of $\overline{RT}$.
    b. Prove that the perpendicular bisector passes through vertex S.

11. The vertices of $\triangle ABC$ are $A(-4, 1)$, $B(2, 13)$, and $C(10, 9)$.
    a. Find the slope of $\overline{AB}$.
    b. Write an equation of the line that passes through the midpoint, M, of $\overline{BC}$ and is parallel to $\overline{AB}$.
    c. If the line determined in part b intersects side $\overline{AC}$ at point D, the ratio of the length of $\overline{DM}$ to the length of $\overline{AB}$ is
    (1) 1:1 (2) 1:2 (3) 1:3 (4) 1:4

12. The vertices of right triangle ABC are $A(3, 3)$, $B(7, 7)$, and $C(7, -1)$.
    a. Write an equation of the line which passes through B and is parallel to $\overline{AC}$.
    b. If circle O is circumscribed about $\triangle ABC$, find the coordinates of O.

13. Given points $A(2, 2)$ and $B(6, 3)$.
    a. Find the coordinates of A', the image of A after a dilation of constant 4 with respect to the origin. Write an equation of $\overleftrightarrow{AA'}$.
    b. Find the coordinates of B', the image of B after a reflection in $\overleftrightarrow{AA'}$.
    c. Show that $ABA'B'$ is *not* a parallelogram.

14. The vertices of $\triangle PQR$ are $P(8, 6)$, $Q(-1, 13)$, and $R(5, -5)$. The median drawn from P intersects $\overline{QR}$ at point M.
    a. Write an equation of $\overline{PM}$.
    b. Using the methods of coordinate geometry, prove that $\overline{PM}$ is perpendicular to $\overline{QR}$.
    c. Explain why the median and the altitude to side $\overline{QR}$ of $\triangle PQR$ coincide.

15. Given $\triangle ABC$ with vertices $A(3, -1)$, $B(7, 3)$, and $C(-1, 7)$, and $\overline{CD}$ is the altitude to $\overline{AB}$.
    a. Write an equation of the line that contains altitude $\overline{CD}$.
    b. Find the coordinates of the midpoint of $\overline{AB}$. Show that altitude $\overline{CD}$ intersects $\overline{AB}$ at its midpoint.

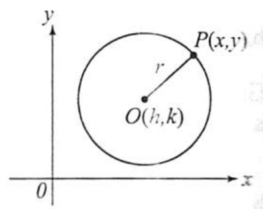
16. The coordinates of the vertices of $\triangle ABC$ are $A(-6, -8)$, $B(6, 4)$, and $C(-6, 10)$.
    a. Write an equation of the altitude from C to $\overline{AB}$.
    b. Write an equation of the altitude from B to $\overline{AC}$.
    c. Find the x-coordinate of the point of intersection of the two altitudes in parts a and b.

17. The coordinates of the vertices of $\triangle NYC$ are $N(-2, 9)$, $Y(6, 3)$, and $C(4, -7)$.
    a. Write an equation of the line that joins the midpoints of sides $\overline{NY}$ and $\overline{NC}$.
    b. Write an equation of $\overleftrightarrow{YC}$.
    c. Show by means of coordinate geometry that the lines determined in parts a and b are parallel.

9.5 GENERAL EQUATION OF A CIRCLE

KEY IDEAS

In the coordinate plane, the locus of points at a fixed distance of r units from point $O(h, k)$ is a circle centered at (h, k) with radius r. Applying the distance formula:



$$\sqrt{(x-h)^2+(y-k)^2} = r$$

Squaring both sides gives a general equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Equation of a Circle: Center–Radius Form

The equation $(x - h)^2 + (y - k)^2 = r^2$ describes a circle whose center is at (h, k) with radius r. If the circle is centered at the origin, then $h = k = 0$ and the equation of the circle simplifies to $x^2 + y^2 = r^2$.

- If the center of a circle is at (2,–1) and its radius is 5, then an equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = -1$, and $r = 5$. Making the substitutions gives $(x - 2)^2 + (y - [-1])^2 = 5^2$ or, equivalently, $(x - 2)^2 + (y + 1)^2 = 25$.
- The center and radius of a circle can be read from its equation. By rewriting the equation $(x - 3)^2 + (y + 4)^2 = 36$ in center–radius form, you can determine that $h = 3$, $k = -4$, and $r = 6$:

$$(x-3)^2 + (y+4)^2 = 36$$
$$(x-3)^2 + (y-(-4))^2 = 6^2$$
$$\downarrow \qquad\qquad \downarrow \qquad \downarrow$$
$$(x-h)^2 + (y-k)^2 = r^2$$

Thus, the center of this circle is (3, –4) and its radius is 6.

Example 1

The coordinates of the endpoints of diameter $\overline{AB}$ of circle O are $A(1, 2)$ and $B(-7, -4)$. Find an equation of circle O.

Solution:

- To find the coordinates (h, k) of the center of the circle, use the midpoint formula:

$$h = \frac{1+(-7)}{2} = \frac{-6}{2} = -3 \quad \text{and} \quad k = \frac{2+(-4)}{2} = \frac{-2}{2} = -1$$

The coordinates of the center of circle O are, therefore, (–3, –1).
- To find the radius length, use the distance formula to find the distance from the center of the circle to any point on the circle, such as point A. If $(x_1, y_1) = A(1, 2)$ and $(x_2, y_2) = O(-3, -1)$, then

$$OA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-3-1)^2+(-1-2)^2}$$
$$= (-4)^2 + (-3)^2$$
$$= \sqrt{16+9}$$
$$= 5$$