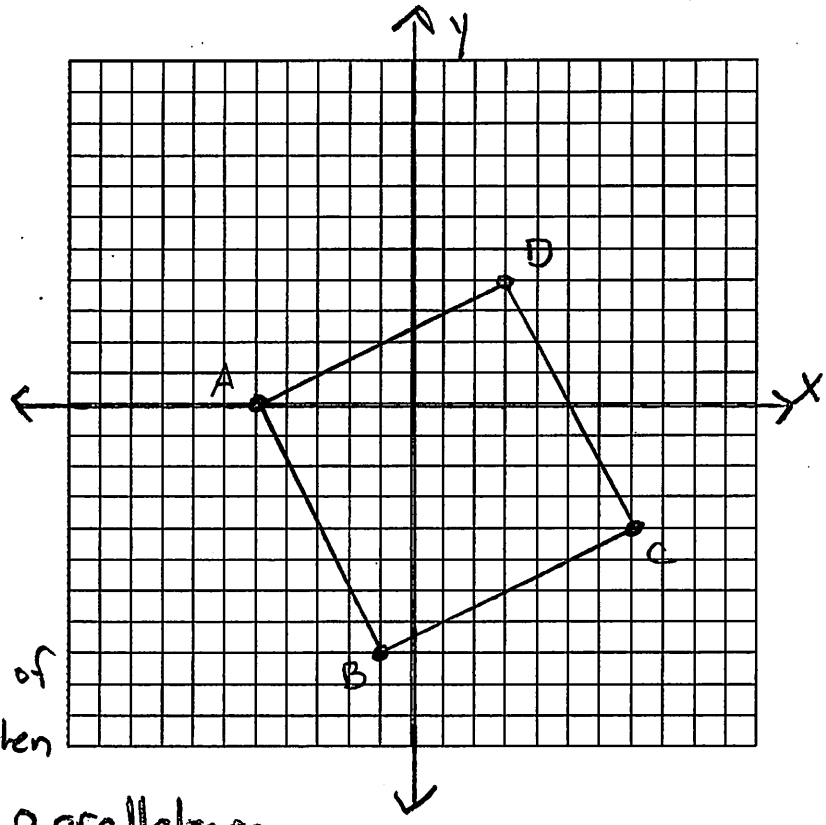
Additional Coordinate Geometry Proof Practice Name Key

1. Given: Quadrilateral *ABCD* with *A*(–5, 0), *B*(–1, –8), *C*(7, –4), *D*(3, 4).
Prove: *ABCD* is a rectangle.



$m_{\overline{AB}} = -2$ $m_{\overline{AD}} = \frac{1}{2}$

$m_{\overline{DC}} = -2$ $m_{\overline{BC}} = \frac{1}{2}$

$\overline{AB} \parallel \overline{DC}$ $\overline{AD} \parallel \overline{BC}$

Same slopes

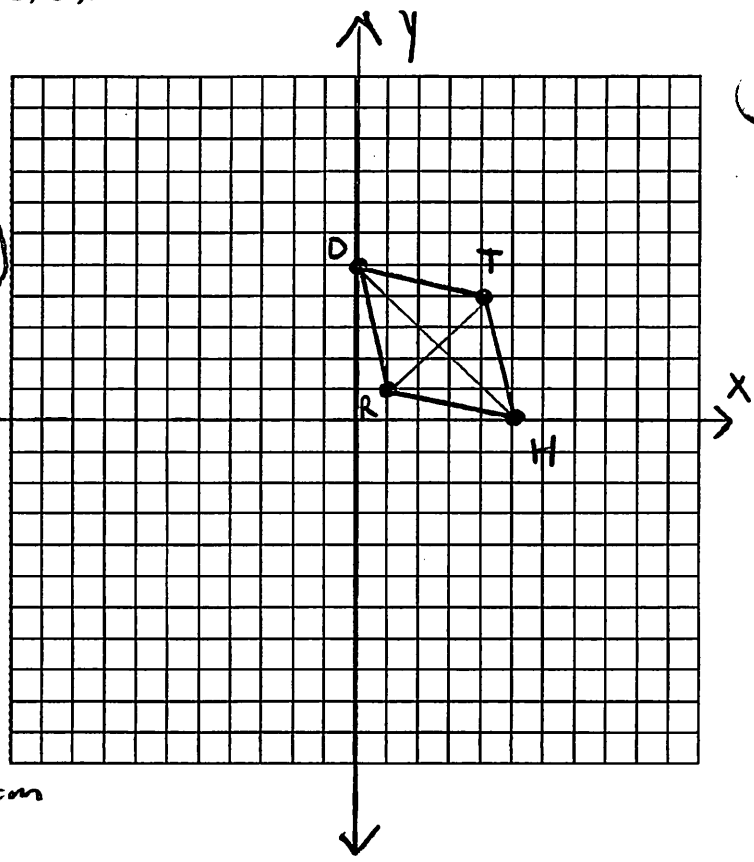
∴ If opposite sides of ABCD are ∥ then the quad is a parallelogram

$\overline{AB} \perp \overline{BC}$ because the slopes are neg recips

↳ $\angle B$ is a right $\angle$

∴ If parallelogram ABCD has 1 right $\angle$ then ABCD is a rectangle

2. Given $R(1, 1)$, $O(0, 5)$, $T(4, 4)$, and $H(5, 0)$.
Prove *ROTH* is a rhombus.



Midpt $\overline{RT}$ $\left(\frac{4+1}{2}, \frac{4+1}{2}\right) \rightarrow \left(\frac{5}{2}, \frac{5}{2}\right)$

Midpt $\overline{OH}$ $\left(\frac{5+0}{2}, \frac{0+5}{2}\right) \rightarrow \left(\frac{5}{2}, \frac{5}{2}\right)$

If the diagonals have the same midpt then they bisect each other

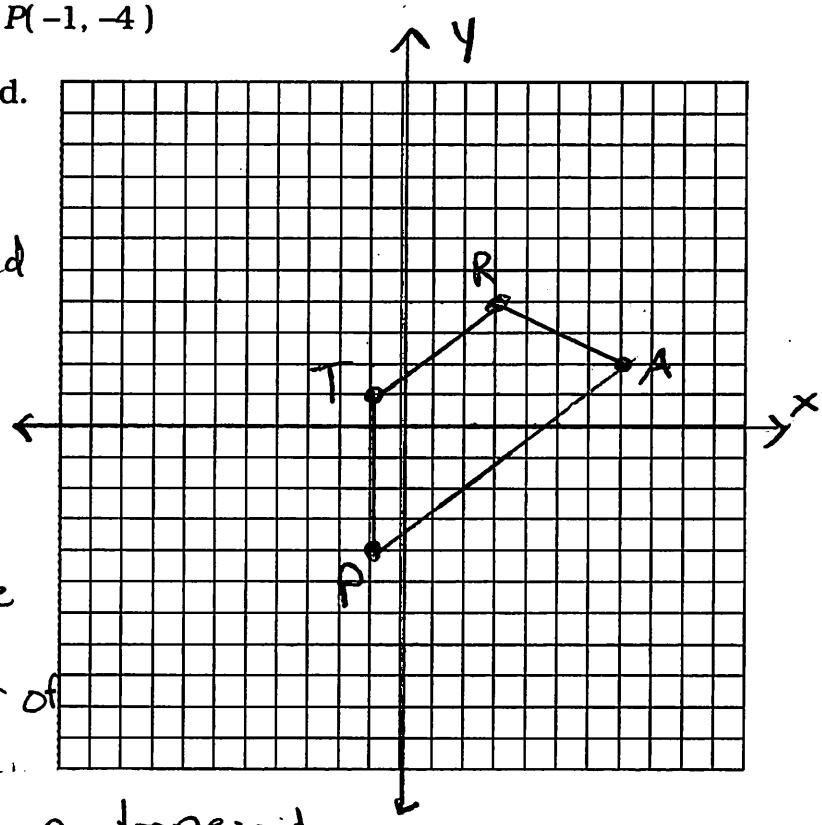
- If the diags of ROTH bisect each other then ROTH is a parallelogram

$m_{\overline{RT}} = 1 \qquad m_{\overline{OH}} = -1$

$\overline{RT} \perp \overline{OH}$ b/c slopes are neg recips

- If parallelogram ROTH has $\perp$ diags. then ROTH is a rhombus

3. Given: $T(-1, 1)$, $R(3, 4)$, $A(7, 2)$, and $P(-1, -4)$
Prove: *TRAP* is a trapezoid.
TRAP is not an isosceles trapezoid.



$m_{\overline{TR}} = \frac{3}{4}$ $\quad m_{\overline{TP}}$ = Undefined

$m_{\overline{PA}} = \frac{3}{4}$ $\quad m_{\overline{RA}} = -\frac{1}{2}$

$\overline{TR} \parallel \overline{PA}$ → same slope

$\overline{TP} \nparallel \overline{RA}$ → not same slope

- If TRAP has a pair of
- ∥ sides and a pair of non-∥ sides, then it is a trapezoid

$d_{\overline{TP}} = 5$ $\quad d_{\overline{RA}} = \sqrt{(7-3)^2+(2-4)^2}$

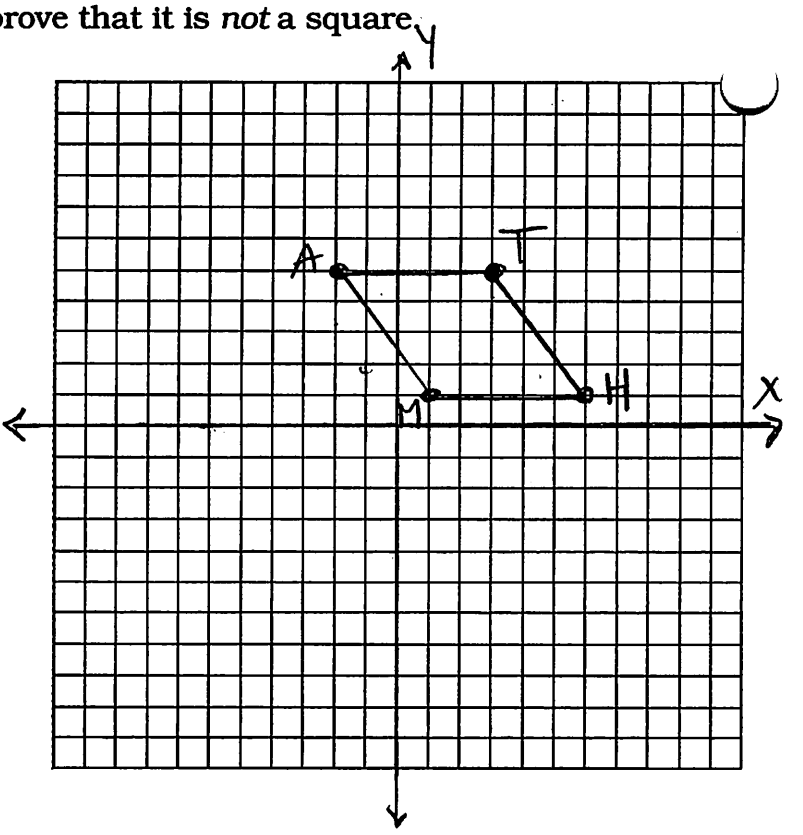
$= \sqrt{4^2+(-2)^2}$

$= \sqrt{20}$

$\overline{TP} \neq \overline{RA}$

- If in trapezoid TRAP the non-∥ sides are
- not $\cong$ then TRAP is not an isos. trapezoid.

4. Quadrilateral *MATH* has coordinates *M*(1, 1), *A*(–2, 5), *T*(3, 5), and *H*(6, 1). Prove that quadrilateral *MATH* is a rhombus and prove that it is *not* a square.



$d_{\overline{AT}} = 5$ $\quad d_{\overline{MH}} = 5$

$d_{\overline{AM}} = \sqrt{(1+2)^2 + (1-5)^2}$

$= \sqrt{3^2 + (-4)^2}$

$= \sqrt{25} = 5$

$d_{\overline{TH}} = \sqrt{(6-3)^2 + (1-5)^2}$

$= \sqrt{3^2 + (-4)^2}$

$= \sqrt{25} = 5$

$\overline{AT} \cong \overline{MH} \cong \overline{AM} \cong \overline{TH}$

- If MATH has all
- sides $\cong$ then it is a rhombus

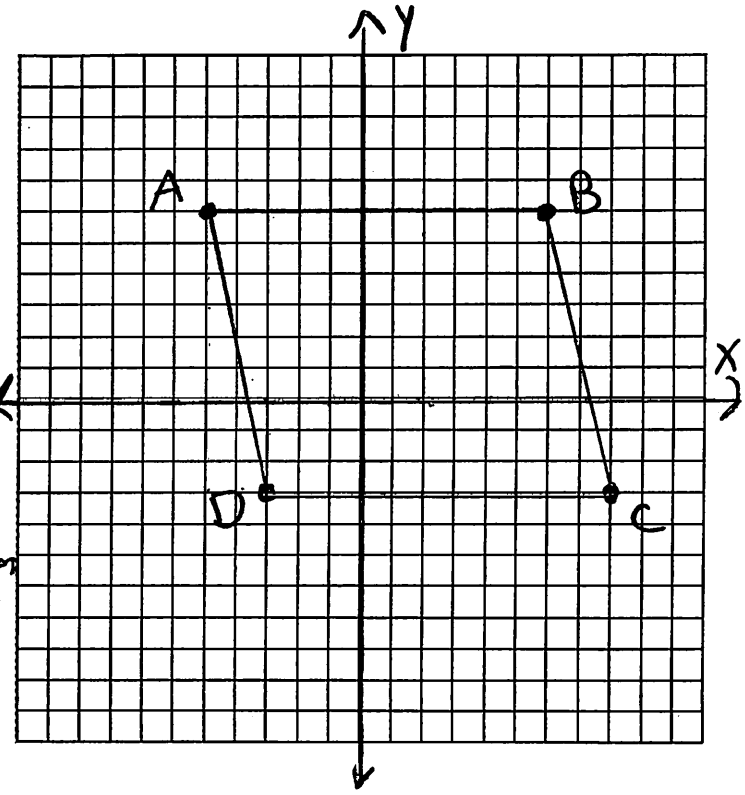
$m_{\overline{AT}} = 0$ $\quad m_{\overline{AM}} = -\frac{4}{3}$

$\overline{AT} \not\perp \overline{AM}$ because slopes are not neg recip.

↳ $\angle A$ is not right

- If rhombus MATH does not have all right
- $\angle$'s then it cannot be a square.

5. Given: Quadrilateral *ABCD* has vertices *A*(–5, 6), *B*(6, 6), *C*(8, –3), and *D*(–3, –3).
Prove: Quadrilateral *ABCD* is a parallelogram but is neither a rhombus nor a rectangle.



$d_{\overline{AB}} = 11$ $\quad d_{\overline{DC}} = 11$

$m_{\overline{AB}} = 0$ $\quad m_{\overline{DC}} = 0$

$\overline{AB} \cong \overline{DC}$ $\quad \overline{AB} \parallel \overline{DC} \rightarrow$ some slope

- If quad ABCD has a $\cong$ and $\parallel$ pair of sides then ABCD is a parallelogram

$m_{\overline{AD}} = -\frac{9}{2}$ $\quad \overline{AB} \not\perp \overline{AD}$

because slopes are not neg recip

↳ $\angle A$ is not right

- If parallelogram ABCD does not have all right $\angle$'s then ABCD is not a rectangle

$d_{\overline{AD}} = \sqrt{(-5+3)^2 + (6+3)^2}$

$= \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{85}$

$\overline{AB} \ncong \overline{AD}$

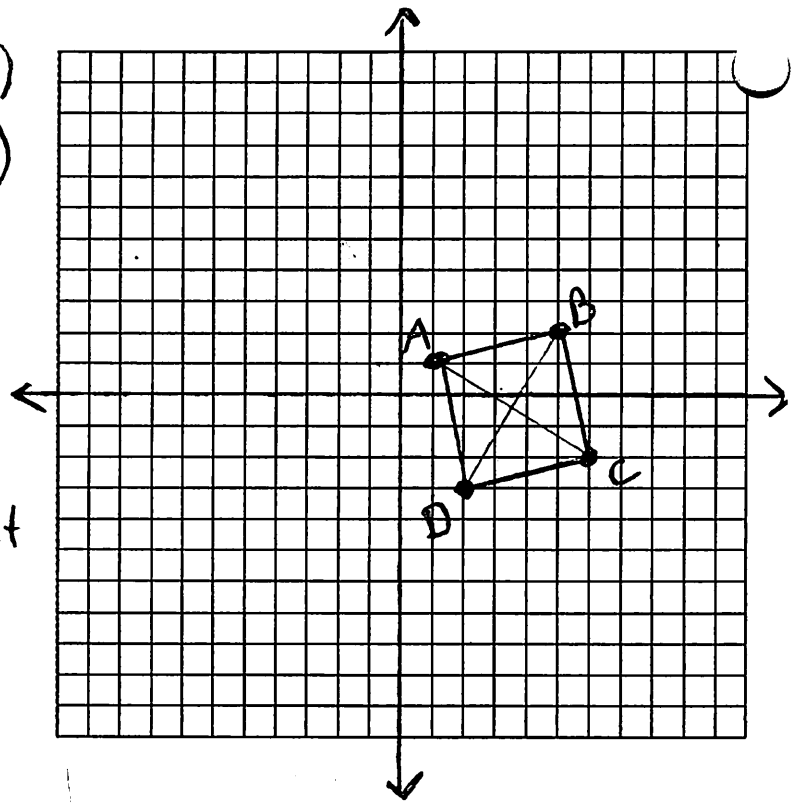
- If parallelogram ABCD does not have all pairs of consecutive sides $\cong$ then ABCD is not a rhombus

6. Quadrilateral *ABCD* has vertices *A*(1, 1), *B*(5, 2), *C*(6, –2), and *D*(2, –3). Classify the quadrilateral.

Midpt $\overline{AC}$ $\left(\frac{6+1}{2}, \frac{-2+1}{2}\right) \rightarrow \left(\frac{7}{2}, -\frac{1}{2}\right)$

Midpt $\overline{BD}$ $\left(\frac{5+2}{2}, \frac{2-3}{2}\right) = \left(\frac{7}{2}, -\frac{1}{2}\right)$

$\overline{AC}$ and $\overline{BD}$ bisect each other b/c they have the same midpt.



- If ABCD have diags that bisect each other then its a parallelogram

$d_{\overline{AC}} = \sqrt{(6-1)^2+(-2-1)^2}$ $d_{\overline{BD}} = \sqrt{(5-2)^2+(2+3)^2}$

$= \sqrt{5^2+(-3)^2}$ $= \sqrt{3^2+5^2}$

$= \sqrt{34}$ $= \sqrt{34}$

$\overline{AC} \cong \overline{BD}$

- If parallelogram ABCD has $\cong$ diags then ABCD is a rectangle

$m_{\overline{AC}} = \frac{-3}{5}$ $m_{\overline{BD}} = \frac{5}{3}$

$\overline{AC} \perp \overline{BD}$ b/c the slopes are neg recip.

- If a parallelogram ABCD has $\perp$ diags then ABCD is a rhombus

- If ABCD is a parallelogram, rectangle and rhombus then it must be a Square.